1.3 BASIC PRINCIPLES AND DEFINITIONS

1.30 GENERAL

It is assumed that engineers using this handbook are thoroughly familiar with the basic principles of strength of materials, such as can be found in any standard text book on this subject. A brief summary of such material is presented here for the sake of uniformity and to emphasize certain principles of special importance.

1.31 STRESS

1.310 General. This term as used herein always implies a force per unit area and is a measure of the intensity of the force acting on a definite plane passing through a given point. (See Eqs. 1:1 and 1:2, Sec. 1.21). The stress distribution may or may not be uniform, depending on the nature of the loading condition. For example, tensile stresses as found from Eq. 1:1, (Sec. 1.21) are considered to be uniform, while the bending stress determined from Eq. 1:3 (Sec. 1.21) refers to the stress at a point located at a distance "y" from the neutral axis. Obviously the stress over the cross section of a member subjected to bending is not uniform. Likewise the shear stresses caused by a shearing load are not uniform (Eq. 1:4 gives the average stress).

1.311 Normal and Shear Stresses. The stresses acting at a point in any stressed member can be resolved into components acting on planes through the point.

The normal and shear stresses acting on any particular plane are the stress components perpendicular and parallel, respectively, to the plane. A simple conception of these stresses is that normal stresses tend to pull apart (or press together) adjacent particles of the material, while shear stresses tend to cause such particles to slide on each other.

1.32 STRAIN

1.320 Axial Strain. This term refers to the elongation, per unit length, of a member or portion of a member in a stressed condition. (See Eq. 1:11, Sec. 1.23). The term "strain" should not be used in place of the terms "elongation" and "deflection".

1.321 Lateral Strain. The axial strain of a member is always accompanied by a lateral strain of opposite sign. The ratio of the lateral strain to the axial strain is called Poisson's ratio and is designated as μ. The value of μ, is usually between 0.25 and 0.33 for steel and the aluminum alloys.

1.322 Shearing Strain. If a square element of uniform thickness is subjected to pure shear there will be a displacement of each side of the element relative to the opposite side. The shearing strain is obtained by dividing this displacement by the distance between the sides of the element. It should be noted that shearing strain is obtained by dividing a displacement by a distance at right angles to the displacement whereas axial strain is obtained by dividing the deformation by a length measured in the same direction as the deformation.

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1.33 TENSILE PROPERTIES

1.330 General. When a specimen of a certain material is tested in tension, it is customary to plot the results of such a test as a "stress-strain diagram". This diagram forms the basis for most strength specifications and should be thoroughly understood and frequently applied by all engineers. Typical tensile diagrams, not to scale, are shown in Fig. 1-1, Page 1-12. It should be noted that the strain scale is non-dimensional, while the stress scale is in pounds per square inch. The important physical properties which can be shown on the stress-strain diagram are discussed in the following sections.

1.331 Modulus of Elasticity (E). Referring to Fig. 1-1a (for materials such as plain low carbon steel), it will be noted that the first part of the diagram is substantially a straight line. This indicates a constant ratio between stress and strain over that range. The numerical value of the ratio is called the MODULUS OF ELASTICITY, denoted by "E". It will be noted that E is the slope of the straight portion of the stress-strain diagram and is determined by dividing the stress (in pounds per square inch) by the strain (which is non-dimensional) (See Eq. 1:12, Sec. 1.23). Therefore, "E" has the same dimension as a stress; in this case pounds per square inch. A useful conception of E is "the stress at which the member would have elongated a distance equal to its original length (assuming no departure from the straight portion of the stress-strain diagram)." This can be easily understood from Eq. 1:12,(Sec. 1.23) by considering that $\delta = L$ in Eq. 1:11, making the strain "e" equal to 1.0.

Other moduli that are often of interest are the tangent modulus E_t, and the secant modulus E_s. The tangent modulus is the slope of the stress-strain diagram at a point corresponding to a given stress while the secant modulus is the slope of a line drawn through the same point and the origin.

Alclad aluminum alloys have two separate modulus values, as indicated in the typical curve presented in Fig. 1-1c. The initial modulus is the same as for the other aluminum alloys, and holds only up to the proportional limit of the relatively soft Alclad covering. Immediately above this point there is a short transition range and the material then exhibits a secondary modulus up to the proportional limit of the stronger core material. This secondary modulus is the slope of the second straight line portion of the diagram. Both values of the modulus are based on the gross area of the piece, core plus covering.

1.332 Tensile Proportional Limit (F_{tp}). Since it is practically impossible to determine the stress at which the stress-strain diagram begins to depart from a straight line, it is customary to assign a small value of permanent strain for this purpose. In this handbook the limit of proportionality will be taken as the stress at which the stress-strain diagram departs from a straight line by a strain of 0.0001. This property or characteristic of a material gives an indication of the type of stress-strain diagram which applies in the working range. It also indicates the stress beyond which the standard value of E cannot be accurately applied. This is of special interest in the analysis of redundant structures.

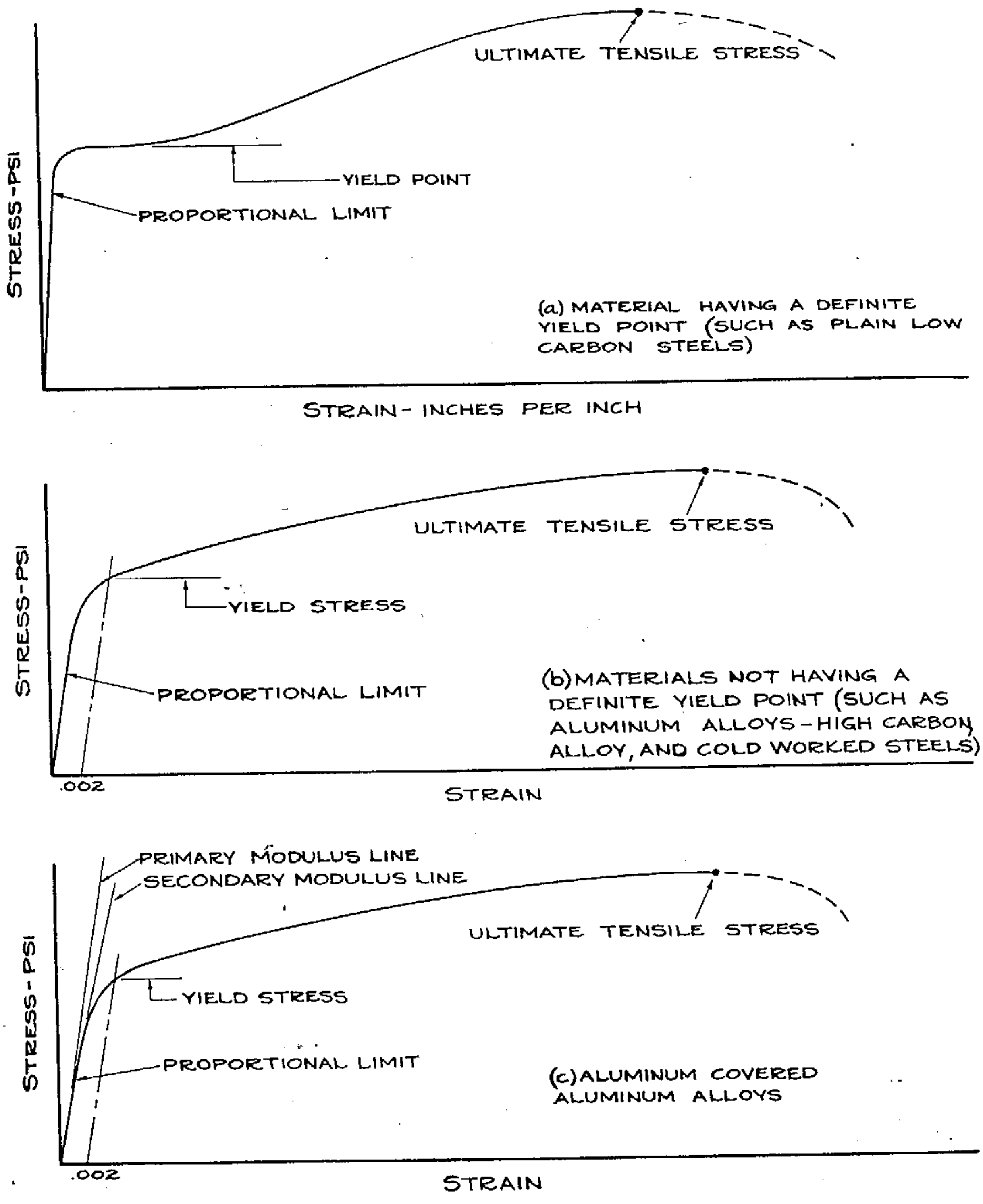

FIG. 1-1. TYPICAL TENSILE STRESS-STRAIN DIAGRAMS (NOT TO SCALE)

1.333 Tensile Yield Stress (F_{ty}). The stress-strain diagrams for plain low carbon steels show a sharp break at a stress considerably below the ultimate tensile stress. At this critical stress the material elongates considerably with little or no increase in stress (See Fig. 1-1a). The stress at which this takes place is referred to as the yield point. Non-ferrous metals, high carbon steels, alloy steels, and cold worked steels do not show this sharp break but yield more gradually so that there is no definite yield point. This condition is illustrated in Fig. 1-1b. Since permanent deformations of any appreciable amount are undesirable in most structures, it is customary to adopt an arbitrary amount of permanent strain that is considered admissable for general purposes. The value of this strain has been established by material testing engineers as 0.002, and the corresponding stress is called the yield stress. For practical purposes this may be determined from the stress-strain diagram by drawing a line parallel to the straight or elastic portion of the curve through a point representing zero stress and 0.002 strain. (See Fig. 1-1). The yield stress is taken as the intersection of this straight line with the stress-strain curve. Obviously the yield stress so determined will coincide with the yield point when the latter is well defined, as shown in Fig. 1-1a.

1.334 Ultimate Tensile Stress (F_{tu}). Figure 1-1 shows how the ultimate tensile stress is determined from the stress-strain diagram. It is simply the stress at the maximum load reached in the test. It should be noted that all stresses are based on the original cross-sectional area of the test specimen, without regard to the lateral contraction of the specimen which actually occurs during the test. The ultimate tensile stress is commonly used as a criterion of the strength of the material, but it should be borne in mind that most modern aircraft structures have relatively few members which are critical in tension; consequently, other strength properties may often be more important.

1.34 COMPRESSIVE PROPERTIES.

1.340 General. The results of compression tests can be plotted as stress-strain diagrams similar to those shown in Fig. 1-1 for tension. The preceding remarks (with the exception of those pertaining to ultimate stress) concerning the specific tensile properties of the material apply in a similar manner to the compressive properties. It should be noted that the moduli of elasticity in tension and compression are approximately equal for most of the commonly used structural materials. Special considerations concerning the ultimate compressive stress are taken up in the following section.

1.341 Ultimate Compressive Stress (F_{cu}). It is difficult to discuss this property without reference to column action. Almost any piece of material, unless very short tends to buckle laterally as a column under compressive loadings, and the load at failure usually depends on the relation of the length of the piece to its cross-sectional dimensions. Column failure cannot occur, however, when a piece is very short in comparison with its cross-sectional dimensions, or when it is restrained laterally by external means. Under these conditions some materials such as stone, wood, and a few metals will fail by fracture, thus giving a definite value for the ultimate compressive stress. Most metals, however, are so ductile that no fracture is encountered in compression. Instead of fracturing, the material yields and swells out, so that the increasing area continues to support the increasing load. It is almost impossible to select a value for the ultimate compressive stress of such materials without having some arbitrary criterion. For wrought metals it is common practice to assume that the ultimate compressive stress is equal to the ultimate tensile stress. For some cast metals which are relatively weak in tension, an ultimate compressive stress higher than the ultimate tensile stress may be obtained from tests on short compact specimens. When tests are made on such specimens having an L/ρ approximately equal to 12, the ultimate stress so obtained is called the "block" compressive stress.

1.35 SHEAR PROPERTIES

1.350 General. The results of torsion tests on round tubes or round solid sections are sometimes plotted as torsion stress-strain diagrams. The modulus of elasticity in shear as determined from such a diagram is a basic shear property. Other properties, such as the proportional limit and ultimate shearing stress, cannot be treated as basic properties because of the "form factor" effects which may occur in such a test.

1.351 Modulus of Elasticity in Shear (G). This property is the ratio of the shearing stress to the shearing strain at low loads, or simply the initial slope of the stress-strain diagram for shear. It is also called the modulus of rigidity. The relation between this property, Poisson's ratio, and the modulus of elasticity in tension, is expressed for homogeneous materials by the following equation:

$$G = \frac{E}{2(1+\mu)} \quad \text{- - - - - - - - - - - - - - - - -} \quad (1{:}34)$$

1.352 Proportional Limit in Shear (F_{sp}). This property is of particular interest in connection with formulas which are based on considerations of perfect elasticity, as it represents the limiting value of shearing stress to which these formulas can be accurately applied. As previously noted, this property cannot be determined directly from torsion tests. The results of research at the National Bureau of Standards show that the ratio of the proportional limit in shear to the proportional limit in tension can be assumed to be approximately 0.55 for the commonly used structural materials.

1.353 Yield and Ultimate Stresses in Shear. These properties, as usually obtained from torsion tests, are not strictly basic properties as they will depend on the shape of the test specimen. In such cases they should be treated as moduli and should be used only with specimens which are geometrically similar to those from which the test results were obtained.

1.4 TYPES OF FAILURES

1.40 GENERAL

In the following discussion the term "failure" will usually denote actual rupture of the member, or the condition of the member when it has just attained its maximum load.

1.41 MATERIAL FAILURES

1.410 General. Fracture of a material may occur by either a separation of adjacent particles across a section perpendicular to the direction of loading, or by a sliding of adjacent particles along other sections. In some cases the mechanism of failure includes both of these actions. For instance, in a simple tension test sliding action along inclined sections may occur first with a consequent reduction in the cross sectional area of the specimen. This may result in strain hardening of the material so that the resistance to sliding is increased, and the final failure may occur by separation of the material across a section perpendicular to the direction of the loading.

1.411 Direct Tension or Compression. This type of failure is associated with the ultimate tensile or compressive stress of the material. For compression it can apply only to members having large cross-sectional dimensions as compared to the length in the direction of the load. See also Sec. 1.341.

1.412 Shear. Pure shear failures are usually obtained only when the shear load is transmitted over a very short length of the member. This condition is approached in the case of rivets and bolts. In cases where the ultimate shear stress is relatively low a pure shear failure may result, but in general a member subjected to a shear load fails under the action of the resulting normal stresses (Eq. 1:10), usually the compressive stresses. The failure of a tube in torsion, for instance, is not usually caused by exceeding the allowable shear stress, but by exceeding a certain allowable normal compressive stress which causes the tube to buckle. It is customary, for convenience, to determine the allowable stresses for members subjected to shear in the form of shear stresses. Such allowable shear stresses are therefore an indirect measure of the stresses actually causing failure.

1.413 Bearing. The failure of a material in bearing may consist of crushing, splitting, or progressive rapid yielding in the region where the load is applied. Failure of this type will depend, to a large extent, on the relative size and shape of the two connecting parts. The allowable bearing stress will not always be applicable to cases in which one of the contacting members is relatively thin. It is also necessary, for practical reasons, to limit the working bearing stress to low values in such cases as joints subjected to reversals of load or in bearings between movable surfaces. These special cases are covered by specific rulings of the procuring or licensing agencies, involving the use of higher factors of safety in most cases.

1.414 Bending. For compact sections not subject to instability, a bending failure can be classed as a tensile or compressive failure caused by exceeding a certain allowable stress in some portion of the specimen. It is customary to determine, experimentally, the "modulus of rupture in bending", which is a stress derived from test results through the use of Eq. 1:3, (Sec. 1.21) in which case M is the value of bending moment which caused failure. If not determined experimentally, the value of the modulus of rupture in bending may be assumed equal to the ultimate tensile stress when instability is not critical. Since it is well known that Eq. 1:3 is based on assumptions which are not always fulfilled at failure, the modulus of failure cannot be considered as the actual stress at the point of rupture. This should be borne in mind in dealing with combined stresses, such as bending and compression, or bending and torsion.

1.415 Failure from Combined Stresses. In combined stress conditions where failure is not due to buckling or instability it is necessary to refer to some theory of failure. The "maximum shear" theory has received wide acceptance as a simple working basis in the case of ductile materials. (Ref. 1). It should be noted that this theory interprets failure as the first yielding of the material, so that any extension of the theory to cover conditions of final rupture must be based on the experience of the designer. The failure of brittle materials under combined stresses can generally be treated by the "maximum stress" theory.

1.416 Failure due to Stress Concentration and Fatigue. The component parts of the airplane structure are subjected to loading conditions and internal stresses of a highly variable character. It is well known that the strength of a material under such conditions is less than that which would be obtained under steady loadings. This phenomenon of the decreased strength of a material under repeated stresses is commonly called fatigue. For engineering purposes a measure of the basic fatigue characteristics of a material is often obtained by an endurance test. In this handbook the endurance limit in bending will be taken as the maximum alternating bending stress that a polished specimen of the material can withstand for a specified number of cycles, as measured in the "rotating beam" type of endurance test. In a similar manner the endurance limit in torsion will be taken as the maximum alternating stress which the material can withstand for a specified number of cycles. Although the absolute values of these endurance limits do not have much direct application in design they are useful as a relative indication of the fatigue characteristics of the various materials of construction.

Rapid changes in the cross section of a structural member will cause local stress concentrations in which the maximum stresses may be greatly in excess of the average stress. These concentrations do not appreciably affect the strength of ductile materials under steady loads because of the equalization of stresses which takes place after the material begins to yield. Under variable stresses, however, such concentrations may cause local stresses in excess of the endurance limit and thus decrease the ultimate strength of the member. Other factors of major importance in this connection are the average stress and the range of the stress variation.

1.42 INSTABILITY FAILURES

1.420 General. Practically all structural members such as beams and columns, particularly those made from thin material, are subject to failure through instability. In general, instability can be classed as: (1) primary, or (2) local. For example, the failure of a tube under compression may occur either through lateral deflection of the tube as a column (primary instability), or by collapse of the tube wall at a stress lower than that required to produce a general column failure. Similarly, an I-beam may fail by a general sidewise deflection of the compression flange, or by local wrinkling of thin outstanding flanges. It is obviously necessary to consider both types of failures unless it is apparent that the critical load for one type is definitely less than that for the other type.

Instability failures may occur in either the elastic range (below the proportional limit) or in the plastic range (above the proportional limit). To distinguish between these two types of action it is not uncommon to refer to them as elastic instability failures and plastic instability failures, respectively. It is important to note that instability failures are not usually associated with the ultimate stresses of the material. This should be borne in mind when correcting test results for material variations. It also has a bearing on the choice of a material for a given type of construction as the "strength-weight ratio" will be determined from different physical characteristics when this type of failure can be expected. For materials which have a very small spread between the proportional limit and the yield stress, the plastic instability type of failure occurs in such a narrow range that it is not of much importance, but in materials which have a considerable spread between these two properties, the plastic instability type of failure may be equally as important as the elastic type.

In studying any structural member it is important to avoid confusion between the different types of failure, particularly where instability is expected to be important. In general, most members should be investigated first from the standpoint of failures of material. They should then be checked separately for their resistance to primary instability failure. Members which are suspected of being weak in resisting local instability should also be checked for this third possible type of failure. Whichever type of failure gives the lowest strength should be used as the criterion in design.

1.421 Instability Failures under Compressive Loadings. Failures of this type are discussed in Sec. 1.5 (Columns) and in Sec. 1.6 (Thin-Walled Sections).

1.422 Bending Instability Failures. Failures of round tubes of usual sizes when subjected to bending are usually of the plastic instability type. In such cases the criterion of strength is the modulus of rupture as derived from test results through the use of Eq. 1:3 (Sec. 1.21). Elastic instability failures of thin-walled tubes having high D/t ratios are treated in later sections.

1.423 Torsional Instability Failures. The remarks of the preceding section apply in a similar manner to round tubes under torsional loading. In such cases the modulus of rupture in torsion is derived through the use of Eq. 1:6 (Sec. 1.21).

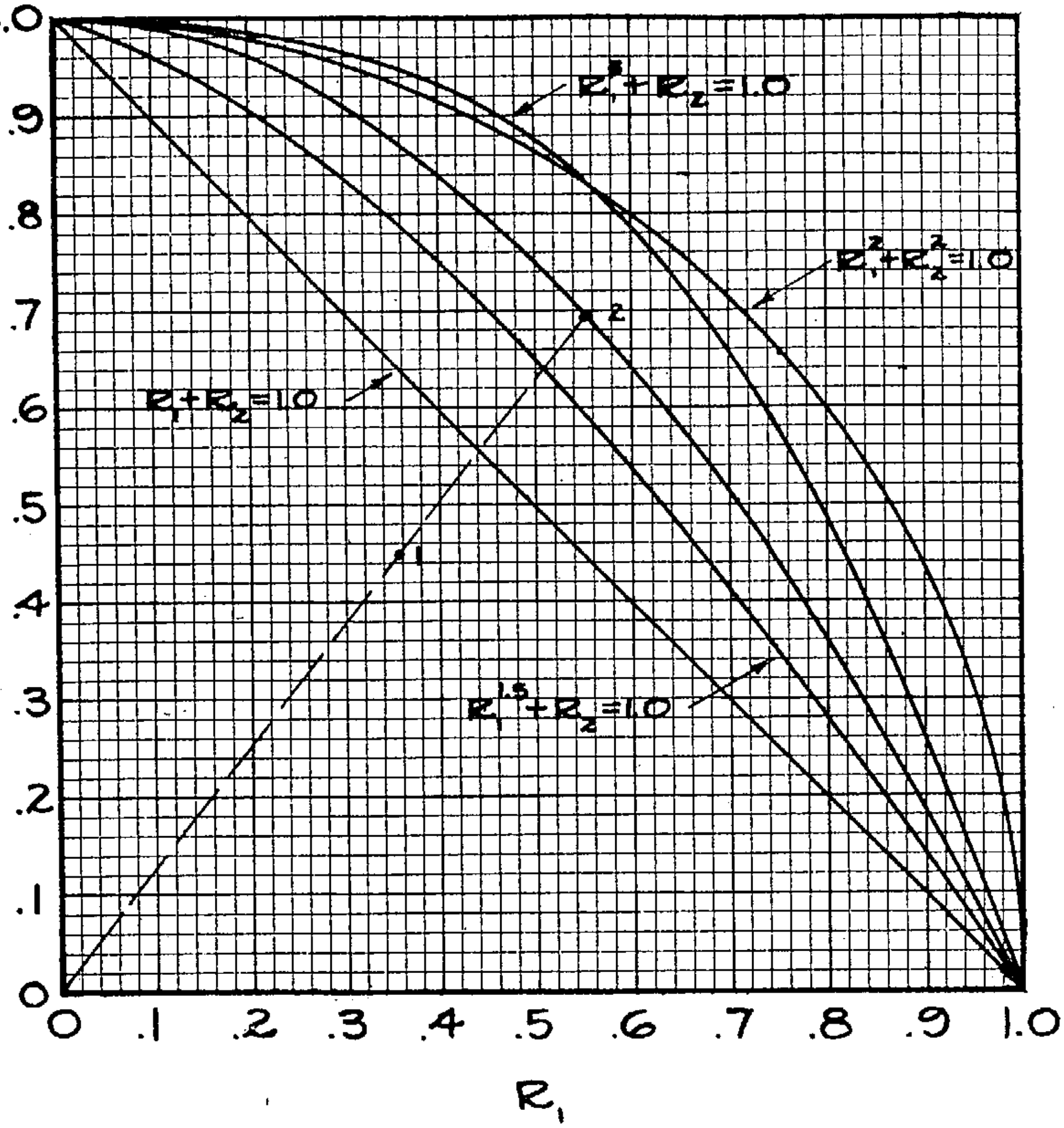

FIG. 1-2. TYPICAL INTERACTION CURVES FOR COMBINED LOADING CONDITIONS

1.424 Failure under Combined Loadings. For combined loading conditions in which failure is caused by buckling or instability, no general theory exists which will apply in all cases. It is convenient, however, to represent such conditions by the use of "stress ratios", which can be considered as non-dimensional coefficients denoting the fraction of the allowable stress or strength which is utilized or which can be developed under special conditions. For simple stresses the stress ratio can be expressed as

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$$R = \frac{f}{F} \text{ - - - - - - - - - - - - - - - - - - } 1.35$$

where f = applied stress

F = allowable stress

Note that the "margin of safety" as usually expressed, is given by the equation:

$$M.S. = \frac{1}{R} - 1.0 \text{ - - - - - - - - - - - - } (1{:}36)$$

Considering the case of combined loadings, the general conditions for failure can be expressed by equations of the following type:

$$R_1^x + R_2^y + R_3^z + \text{ - - - - - } = 1.0 \text{ - - - - - - - - } (1{:}37)$$

In this equation R_1, R_2 and R_3 may denote, for instance, the stress ratios for compression, bending, and shear, and the exponents x, y and z define the general relationship of the quantities. This equation may be interpreted as indicating that failure will occur only when the sum of the stress ratios is equal to or greater than one. An advantage of this method is that the formula yields correct results when only one loading condition is present. Consequently it tends to give good results when any one loading condition predominates. It also permits test data to be plotted in non-dimensional form, which is a decided advantage.

In many cases it is convenient to deal directly with "load ratios" rather than stress ratios. The load ratio is simply the ratio of the applied load to the allowable load and is equal to the corresponding stress ratio.

Considering only two loading conditions, such as bending and torsion, Eq. 1:37 can be plotted as a single interaction curve of R_b against R_s. Likewise, in the case of combined bending and compression, R_c can be plotted against R_b. When all three conditions exist, the equation represents an interaction surface, which can be plotted as a family of curves. Typical curves corresponding to various exponents are shown in Figure 1-2, Page 1-18. The general significance of Eq. 1:37 and Fig. 1-2 is that the addition of a second loading condition will lower the percentage of the allowable stress which may be utilized in the original loading condition. If the exponents approach infinity, the curve of Fig. 1-2 will approach the lines $R_1 = 1.0$ and $R_2 = 1.0$, indicating that the two loading conditions have no effect on each other.

When only two stress-ratios are involved and when the two different applied stresses remain in constant proportion, the margin of safety of the member may be determined from Fig. 1-2 by the following method:

(1) Locate the point on the chart representing the applied values of R_1 and R_2 computed from the applied stresses. (Illustrated as point (1) on Fig. 1-2).

(2) Draw a straight line through this point and the origin (shown as a diagonal dotted line on Fig. 1-2).

(3) Extend this line to intersect the proper stress-ratio curve (corresponding to the condition under consideration) at point (2).

(4) Read the allowable values R_{1a} and R_{2a} as the ordinate and abscissa, respectively, of point (2).

(5) The <u>factor of utilization</u> or <u>strength ratio</u> is obtained as the ratio of the applied to the allowable value of either stress ratio as follows:

$$U = \frac{R_1}{R_{1a}} = \frac{R_2}{R_{2a}} \quad \text{-------------} \quad (1.38)$$

(6) The true margin of safety then can be computed from the following equation:

$$M.S. = \frac{1}{U} - 1 \quad \text{--------------} \quad (1.39)$$

Note that when the following stress ratio expressions are used, the margins of safety can be computed as indicated

For $R_1 + R_2 = 1$,

$$M.S. = \frac{1}{R_1 + R_2} - 1$$

For $R_1{}^2 + R_2{}^2 = 1$

$$M.S. = \frac{1}{\sqrt{R_1{}^2 + R_2{}^2}} - 1$$

Other M.S. formulas can, of course, be determined for the more complicated stress ratio expressions.

The practical application of Eq. 1:37 will be taken up in the following chapters.

1.5 COLUMNS

1.50 GENERAL

A theoretical treatment of columns can be found in standard text books on the strength of materials. The problems confronting the designer include, however, many points which are not well defined by theory and which frequently cause some confusion. These will be taken up in this section. Actual strengths of columns of various types are given in subsequent chapters.

1.51 PRIMARY INSTABILITY FAILURE

1.510 General. A column may fail through primary instability by bending laterally or by twisting about some axis parallel to its own axis. This latter type of primary failure is particularly common to columns having unsymmetrical open sections. The twisting failure of a closed section column is precluded by its inherently high torsional rigidity. Since the available information on twisting instability is somewhat limited it is advisable to conduct tests on all columns subject to this type of failure. A theoretical treatment of primary failure through twisting instability may be found in Reference 2.

1.511 Long Columns. The Euler formula for long columns which fail by lateral bending is given by Eq. 1:22, Sec. 1.27. No explanation of this classical formula need be offered, as its derivation can be found in many standard text books on the strength of materials. The value to be used for the restraint coefficient, c, depends on the degree of end fixation. Definite rules as to the maximum value which may be assumed are given in the specific airworthiness requirements of the Government services. The true significance of the restraint coefficient is best understood by considering the end restraint as modifying the effective column length, as indicated in Eq. 1:22, Sec. 1.27. For a pin-ended column having zero end restraint $c = 1.0$ and $L' = L$. A fixity coefficient of 2 corresponds to a reduction of the effective length to $1/\sqrt{2}$ or .707 times the total length.

1.512 Short Columns. If the length of a column is reduced below a certain critical value, primary bending failure will occur at loads below those predicted by the Euler formula. This is due to a reduction in the effective value of E which is caused by changes in the slope of the stress-strain diagram and by unavoidable eccentricities. In this region the test results show more scatter than in the Euler range and it is customary to adopt an empirical or semi-empirical formula for predicting the allowable column stress. When a definite eccentricity exists, the critical column loads are reduced due to the combined effects of axial load and bending. Special formulas for such cases can be found in standard text books and handbooks.

Although many types of formulas have been devised to cover the short-column range, it has been customary, in aircraft work, to use the Johnson formula for round steel tubes and the straight line formula for round aluminum alloy tubes. Recent tests at the National Bureau of Standards have shown, however, that the Johnson formula does not have the correct shape for round tubes of normalized X-4130 steel.

A modified parabola having an exponent 1.5 (Eq. 1:25, Sec. 1.27) gives a satisfactory representation of the Bureau of Standards test data on round tubes of this material and it has therefore been adopted for this handbook. The Johnson formula (Eq. 1:24, Sec. 1.27) will be used for solid wood struts, round 1025 steel tubes and round heat-treated alloy steel tubes. The straight line formula (Eq. 1:26, Sec. 1.27) will be used for round aluminum alloy tubes.

It will be noted that the above column formulas are of the general form given by Eq. 1:23, Sec. 1.27. For example, the straight line formula is a special case of Eq. 1:23 in which the exponent n is equal to 1.0. In a similar manner the Johnson formula can be obtained from Eq. 1:23 by setting n equal to 2.0. The above equations strictly apply only to round tube sections as they were derived from tests on such sections. In many cases, however, they will be found to be satisfactory for sections of other shapes when local instability is not critical.

Short column failure can also be expressed by the modified Euler formula in which the elastic modulus is replaced by an effective moculus, E', as in the following equation:

$$F_c = \frac{\pi^2 E'}{(L'/\rho)^2} \qquad (1:40)$$

Although this equation does not have much practical importance in determining the short column curve, it is of particular interest in connection with the determination of the effective modulus which can be used to compute local instability stresses. The value of the effective modulus at any given compressive stress, F_c, can be determined by solving Eq. 1:40 for E', after substituting F_c and the corresponding L'/ρ as obtained from the basic column curve for primary failure.

1.513 Column Yield Stress (F_{co}). The upper limit of the allowable column stress for primary failure is called the column yield stress and will be designated F_{co}. It can be determined by extending the "short-column" curve to a point corresponding to zero length, ignoring any tendency of the curve to rise rapidly or "pick-up" for very short lengths. The short-column curve used in determining F_{co} should be obtained from tests on specimens having geometrical proportions such that local failure is precluded except for very low values of L'/ρ.

When the column yield stress is reached, the walls of the column will tend to buckle unless restrained by extreme shortness, or by the application of lateral restraining forces. In some cases, however, if the specimen has not been allowed to buckle, the stress may be increased considerably above this value. Due to the danger of buckling when the column yield stress is approached, the latter should be considered as the limiting stress for all columns.

The column yield stress is mainly determined by the nature of the compressive stress-strain diagram of the material. When the material has a definite yield point in compression, this value may be assumed for the column yield stress. Few aircraft materials, however, have a sharply defined yield point. In such cases it is usually possible to determine the column yield stress as a function of either the tensile or compressive yield stress. For example, F_{co} for normalized X-4130 round tubes is approximately equal to 1.06 times the tensile yield stress. The column yield stress for other materials is given in later sections.

1.52 NON-DIMENSIONAL COLUMN CURVES FOR PRIMARY FAILURE

1.520 General. On account of the many factors involved it is often difficult to predict the effects of possible material variations on the strength of columns as obtained by tests. When the column failure is definitely of the primary bending type it is advisable to plot the test results with non-dimensional coefficients, such as are employed in Reference 3. The following coefficients will be adopted for this purpose:

R_a = allowable stress ratio

$$= F_c/F_{co} \quad \text{------------------} \quad (1:41)$$

where F_c = allowable column stress.

F_{co} = column yield stress.

B = slenderness ratio factor

$$= \frac{L'/\rho}{\pi\sqrt{E/F_{co}}} \quad \text{------------------} \quad (1:42)$$

$L' = L/\sqrt{c}$ (See Eq. 1:22 Sec. 1.27).

The slenderness ratio factor can be considered as the ratio between the effective slenderness ratio (L'/ρ) and the (L'/ρ) at which the Euler stress for a pin-ended column would equal F_{co}. Thus, when $B = 2$, the Euler stress F_{ce} would equal $1/4\ F_{co}$, or R_a would be .25 (since the Euler stress varies inversely as the square of L'/ρ).

1.521 Typical Column Curves. Typical column curves plotted in terms of these non-dimensional coefficients are illustrated in Fig. 1-3, Page 1-24. It will be noted that the Johnson parabolic curve is tangent to the Euler curve at a value of $R_a = .5$; that is, the Euler formula will not apply when it gives stresses higher than half the column yield stress. It is also convenient to know that the stresses given by the 1.5 parabolic formula and the straight line formula are equal to those given by the Euler formula at values of R_a equal to .4286 and .333 respectively.

1.53 LOCAL INSTABILITY FAILURE

1.530 General. Columns may fail by a local collapse of the wall at a stress below the primary failure stress. The general equation for the local failure of round tubes is given in the following section. The local failure of columns having cross sections other than those of round tubes is discussed in Sec. 1.54.

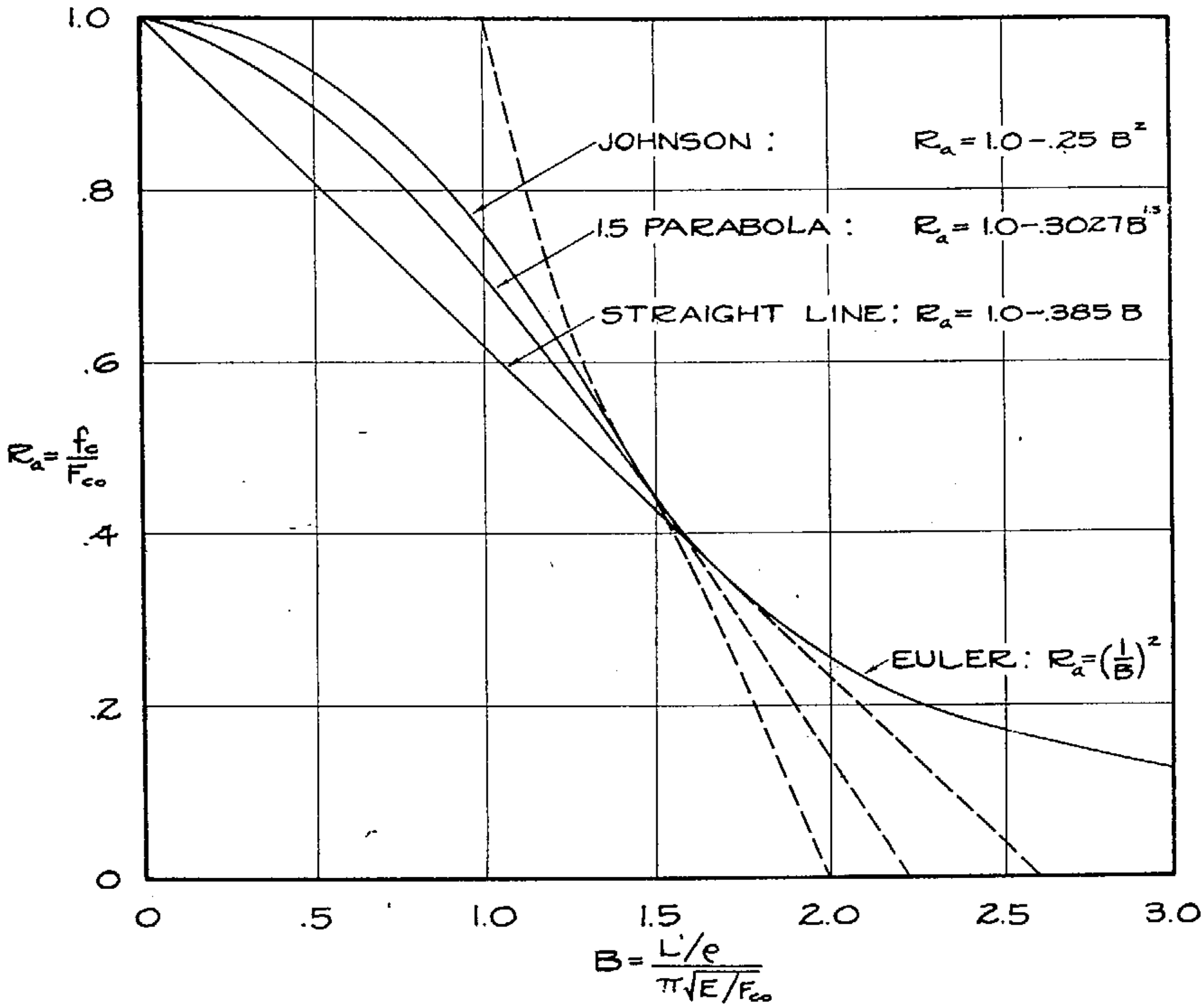

FIG. 1-3. VARIOUS COLUMN CURVES IN NON-DIMENSIONAL FORM

1.531 Crushing or Crippling Stress (F_{cc}). The upper limit of the allowable column stress for local failure is called the crushing or crippling stress and is designated F_{cc}. The crushing stresses of round tubes subject to plastic failure generally can be expressed by a modified form of the equation for the buckling of a thin-walled cylinder in compression (see Sec. 1.630) as given below:

$$F = \frac{K\sqrt{EE'}}{D/t} \qquad (1{:}43)$$

The effective modulus E' can be determined from the basic column curve for primary failure by the method given in Sec. 1.512. As the value of the effective modulus corresponds to a given value of stress it usually is convenient to: (1) assume a value of F_{cc}; (2) compute the corresponding value of E'; (3) substitute these values into Eq. 1:43 and solve for D/t. This latter value is the D/t at which crushing will occur at the assumed stress. Values of the constant K must be determined empirically. As noted above, Eq. 1:43 applies to plastic failure; i.e., for stresses above the proportional limit. In the case of thin-walled tubes which fail locally at stresses below the proportional limit, the initial eccentricities are likely to be larger relatively and the constant should be suitably reduced.

1.54 COLUMNS OF UNCONVENTIONAL CROSS SECTION

1.540 General. In the case of columns having unconventional cross sections which are particularly subject to local instability, it is necessary to establish the curve of transition from local to primary failure. In determining the strength curves for such columns, sufficient tests should be made to cover the following points:

1.541 Nature of "Short Column" Curve. The test specimens should cover a range of L'/ρ which will extend to the Euler range, or at least well beyond the values to be used in construction. When columns are to be attached eccentrically in the structure, some tests should be made to determine the effects of eccentricity. This is important particularly in the case of open sections, as the allowable loads may be affected considerably by the location of the point of application of the column load.

1.542 Local Failure. When local failure occurs, the crushing or crippling stress F_{cc} can be determined by extending the "short column" curve for the specific cross section under consideration to a point corresponding to zero L'/ρ. When a family of columns of the same general cross section is used, it is often possible to determine a relationship between F_{cc} and some factor depending on the wall thickness, width, diameter, or some combination of these dimensions. Extrapolations of such data should be avoided by covering an adequate range in the tests.

1.543 REVISED OCT '40 Reduction of Test Results on Aluminum Alloys to Standard. Although there is no completely rational method for correcting the results of compression tests to standard, the use of the correction factors given in Fig. 1-4 is considered satisfactory and is acceptable to the Army, Navy, and the Civil

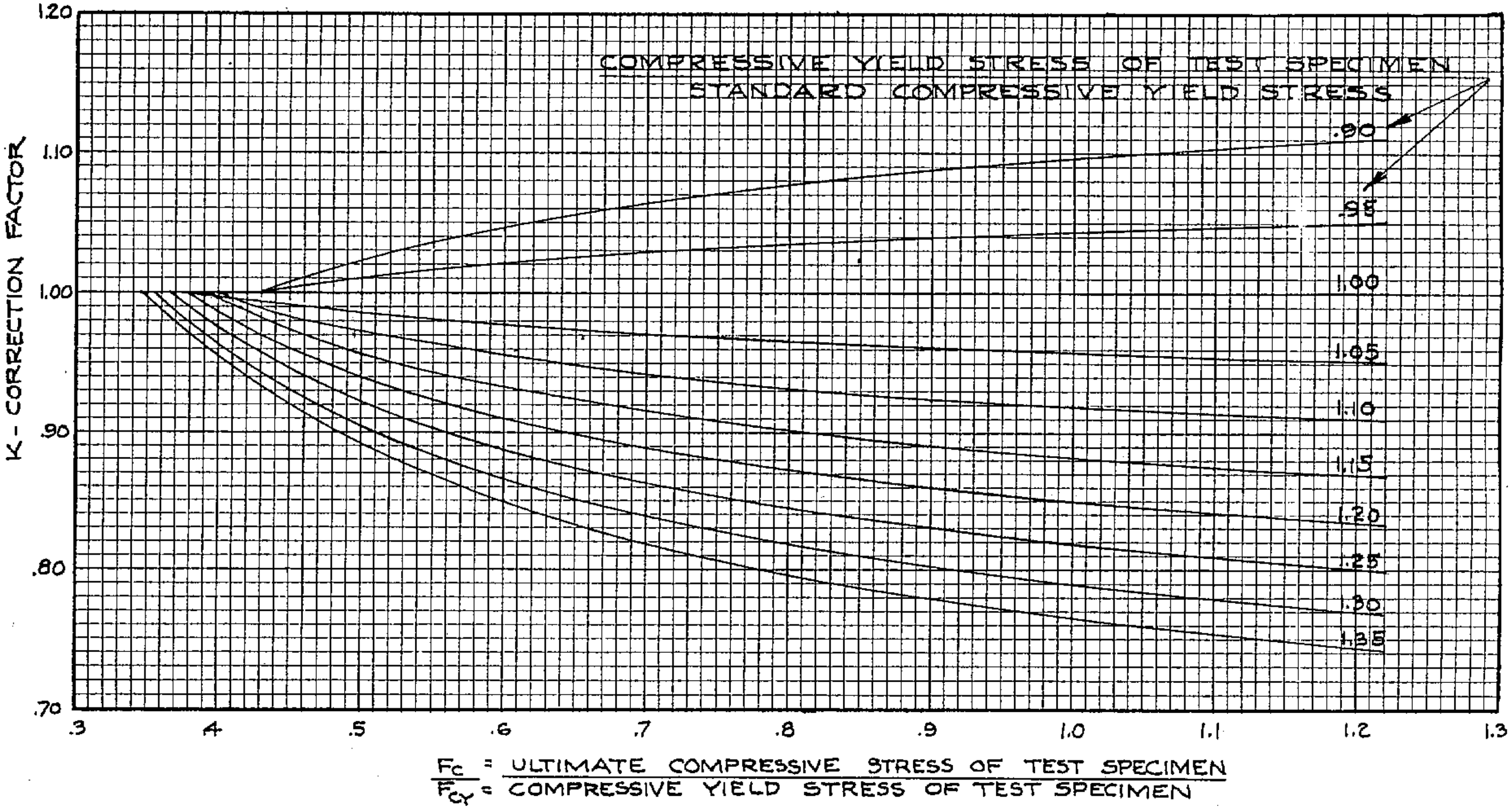

FIG. 1-4 CORRECTION FACTORS FOR COMPRESSION TESTS ON ALUMINUM ALLOY SPECIMENS.

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Aeronautics Administration, for use in connection with tests on aluminum alloys. (Note that an alternative method, acceptable to the Civil Aeronautics Administration only, is given in paragraph 1.544). In using Fig. 1-4, the correction of the test result to standard is made by simply multiplying the stress developed in the test by the factor K. This factor may be considered applicable regardless of the type of failure involved (i.e., column, crushing, or twisting).

For obtaining compressive yield strengths for use in Fig. 1-4, the methods that should be used are:

a. Direct compressive stress-strain measurements of the specimen.
b. In case a compression member is formed from sheet material, and the use of method (a) is not feasible, direct tensile stress-strain measurements should be taken on the original sheet in a direction normal to the length of the compression member. The cross and with-grain yield ratios given in Table I-1 then should be used to compute the compressive yield along the length of the compression member. In case the compression member is manufactured indiscriminately with respect to material grain, the test specimen should be made with the grain parallel to its length.
c. In case neither methods (a) nor (b) are feasible or applicable, it should be assumed that the compressive yield of the specimen is 15 percent greater than the minimum established yield for the material.

TABLE I-1

RELATIONSHIP BETWEEN WITH AND CROSS GRAIN PROPERTIES OF ALUMINUM ALLOY SHEET

Property	For 17ST, 24ST, Alclad 17ST, and Alclad 24ST.	For 24SRT and Alclad 24SRT
Tensile strength (w)	= 1.02 Tensile Str. (x)	= 1.02 Tensile Str.(x)
Tensile yield (w)	= 1.17 Tensile yield (x)	= 1.14 Tensile yield(x)
Compressive yield(w)	= 0.96 Tensile yield (x)	= 0.96 Tensile yield(x)
Compressive yield(x)	= 1.08 Tensile yield (x)	= 1.06 Tensile yield(x)
w = with grain, x = cross grain		