For compression members stretched a controlled amount after heat-treatment or made from sheet with the material grain normal to the direction of the expected load, it is reasonable to increase the value of the "minimum" yield to be used as the basis for the correction of tests results. Such increases will depend upon rigid manufacturing control of the stretching or forming. For stretched material the compressive yields for both the stretched and the unstretched conditions should be determined, and the ratio between the two used to multiply the "minimum" established compressive yield for the "as received" material. For compression members formed from sheet, cross-grain, the "minimum" compressive yield may be taken as 1.08 times the specification tensile yield, if the material is 17ST, Alclad 17ST, 24ST, or Alclad 24ST; and is 1.06 times the specification tensile yield, if the material is 24SRT or Alclad 24SRT.

1.544 ADDED OCT '40 Reduction of Test Results to Standard—Civil Aeronautics Administration Method. The following method of correcting test results to standard is acceptable to the Civil Aeronautics Administration only. It is not restricted as to the type of material involved in the tests. Although the correction of compression test results by means of a factor which is a function of thecompressive yield stresses involved is a more rational procedure, the following method gives reasonable results and obviates the difficulties involved in determining the compressive yield stresses.

a. The correction parameter R is taken as the ratio of the specification (or guaranteed) ultimate tensile stress of the material to the actual ultimate tensile stress of a coupon cut from the test specimen.

b. The column intercept obtained from primary failure tests is corrected by multiplying by R.

c. Local or crushing failure test points are corrected by the following factor:

$$K = R^n,$$

where n is the ratio of the actual local failure test stress, $F_{cc}$, to the ultimate tensile stress of a coupon cut from the test specimen ($n = F_{cc}/F_{tu}$).

d. In cases where it is difficult to determine whether the failure of the specimen is primary or local, the test results are corrected by multiplying by R.

e. Corrections for the variation of the modulus of elasticity from the specification value are considered negligible and are therefore neglected.

It will be noted that the column intercept is corrected by multiplying by R. It is necessary, of course, to employ some method which will adjust suitably short column test points at intermediate values of $L/\rho$ so that the proper shape of the column curve is maintained. The method illustrated in Figure I-5 is considered suitable for this purpose.

As shown in Figure I-5, the test stress is plotted against the actual $L/\rho$ ratio. An Euler curve is then constructed, using a restraint coefficient, c, determined from long column tests using the same end conditions as those used in the short column tests, or using a conservatively estimated value of c. (For these purposes a conservative value of c will be high rather than low. For flat end tests a value of from 3 to 4 appears in order.) $F_1$ is established by extending a line through the test point and tangent to the Euler curve. $F_2 (= RF_1)$ is plotted and a line is drawn through this point and tangent to the Euler curve. The corrected test point then is determined by the intersection of this line with a vertical line through the actual test point. It should be noted that this method of construction can be regarded as applicable regardless of whether or not the short column curve is a straight line. After all of the short column test points have been corrected in this manner, the final design curve can be constructed as an "average" curve through the test points. If desired, a scale for the effective $L/\rho$ can be constructed by multiplying the divisions on the scale of actual $L/\rho$ by $\sqrt{c}$. The allowable stress for any fixity condition can then be found by utilizing the effective $L/\rho$ for that condition. (See equation 5:1.)

The ultimate tensile stress of the test specimen should be determined from a tensile coupon cut from the specimen. This preferably should be done prior to the column tests, although it is believed that there would be little difference in final results if the tensile coupons are cut after the column specimens have been tested. In the latter case, the coupon should be cut close to the point of failure but should not include metal which has been worked severely by the failure. If the column is formed from sheet material, the tensile test coupon can be taken in the direction transverse to rolling. (Dimensions for standard tensile test specimens and method of running standard tensile tests are given in Federal Specification QQ-M-151, obtainable for 5 cents from the Superintendent of Documents, Government Printing Office, Washington, D.C.) It will be noted that this method of determining properties permits the manufacturer to take full advantage of any increase in physical properties which may occur as a result of cold work due to forming, as in the case of certain types of corrugations formed from flat sheet.

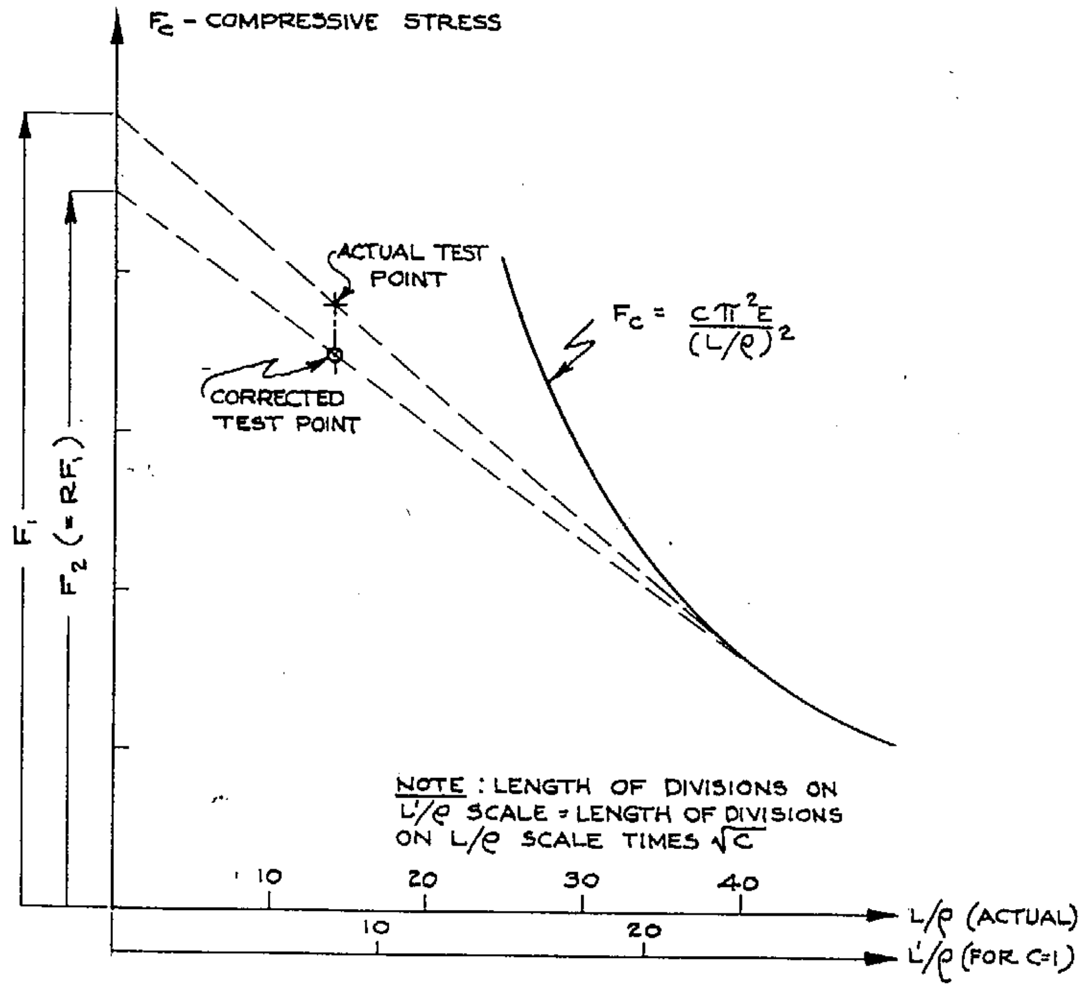


FIG. 1-5 - ILLUSTRATION OF METHOD OF CORRECTING SHORT COLUMN TEST POINTS

(ADDED OCT '40)

## 1.6 THIN-WALLED SECTIONS

### 1.60 GENERAL

The general forms of the equations which apply to thin-walled sections will be given here. In general these equations are based on elastic considerations and for this reason they cannot be accurately applied when they indicate allowable stresses in excess of the proportional limit of the material.

### 1.61 FLAT PANELS

1.610 Buckling Stresses. In References 4 and 5 it is shown that critical stresses for the elastic buckling of flat rectangular panels can be expressed by the following general formula:

$$F_{cr} = KE\,(t/b)^2 \text{ - - - - - - - - - - - - - - (1:44)}$$

Where K is a factor depending on the type of loading, the dimensions of the plate, the edge fixity conditions, and Poisson's ratio.

t = thickness of panel.

b = the loaded side of the panel for compression and bending loads.

= the short side of the panel for shear loads.

Specific forms of Eq. 1:44 for various loading conditions are given in Fig. 1-6, Page 1-32, together with the relevant critical stress factors. These factors are shown for the case of simply supported edges for the fixity conditions realized in typical structures do not usually exceed those corresponding to this type of support. A value of $\mu$ equal to 0.3 was used in deriving these factors.

1.611 Combined Loadings. The conditions for buckling under combined loadings are expressed by general equation 1:37. Specific forms of this equation are given below for flat rectangular plates under various loading conditions.

Compression and Bending

$$R_b^{1.75} + R_c = 1 \text{ - - - - - - - - - - - - - (1:45)}$$

Compression and Shear

$$R_s^{1.5} + R_c = 1 \text{ - - - - - - - - - - - - - (1:46)}$$

Bending and Shear

$$R_b^{2} + R_s^{2} = 1 \text{ - - - - - - - - - - - - - - (1:47)}$$

When compression, shear, and bending loads exist simultaneously the conditions for buckling are represented by the interaction curves of Fig. 1-7, Page 1-33.

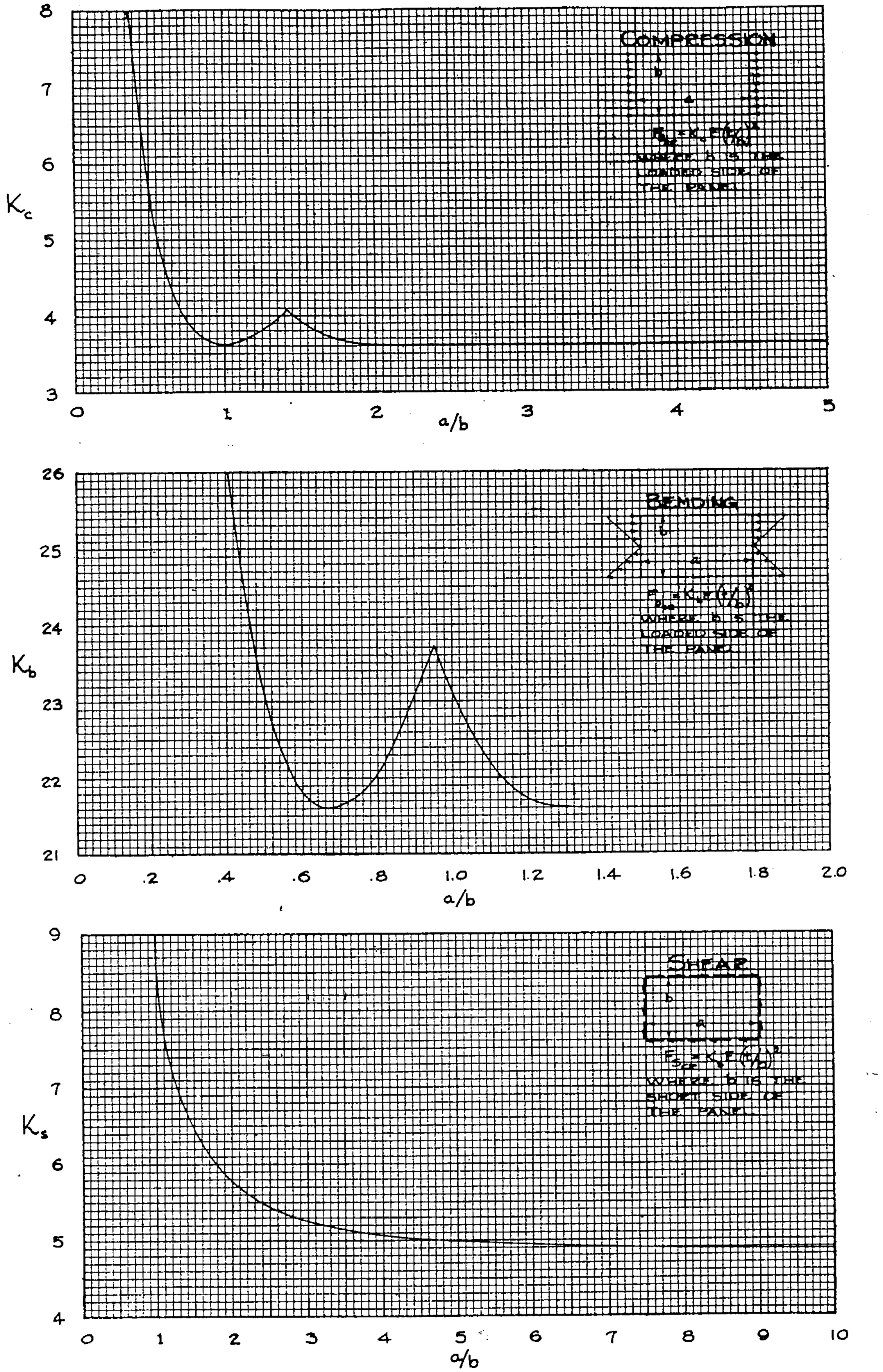


FIG. 1-6 CRITICAL STRESS FACTORS FOR THE ELASTIC BUCKLING OF FLAT RECTANGULAR PANELS HAVING SIMPLY SUPPORTED EDGES

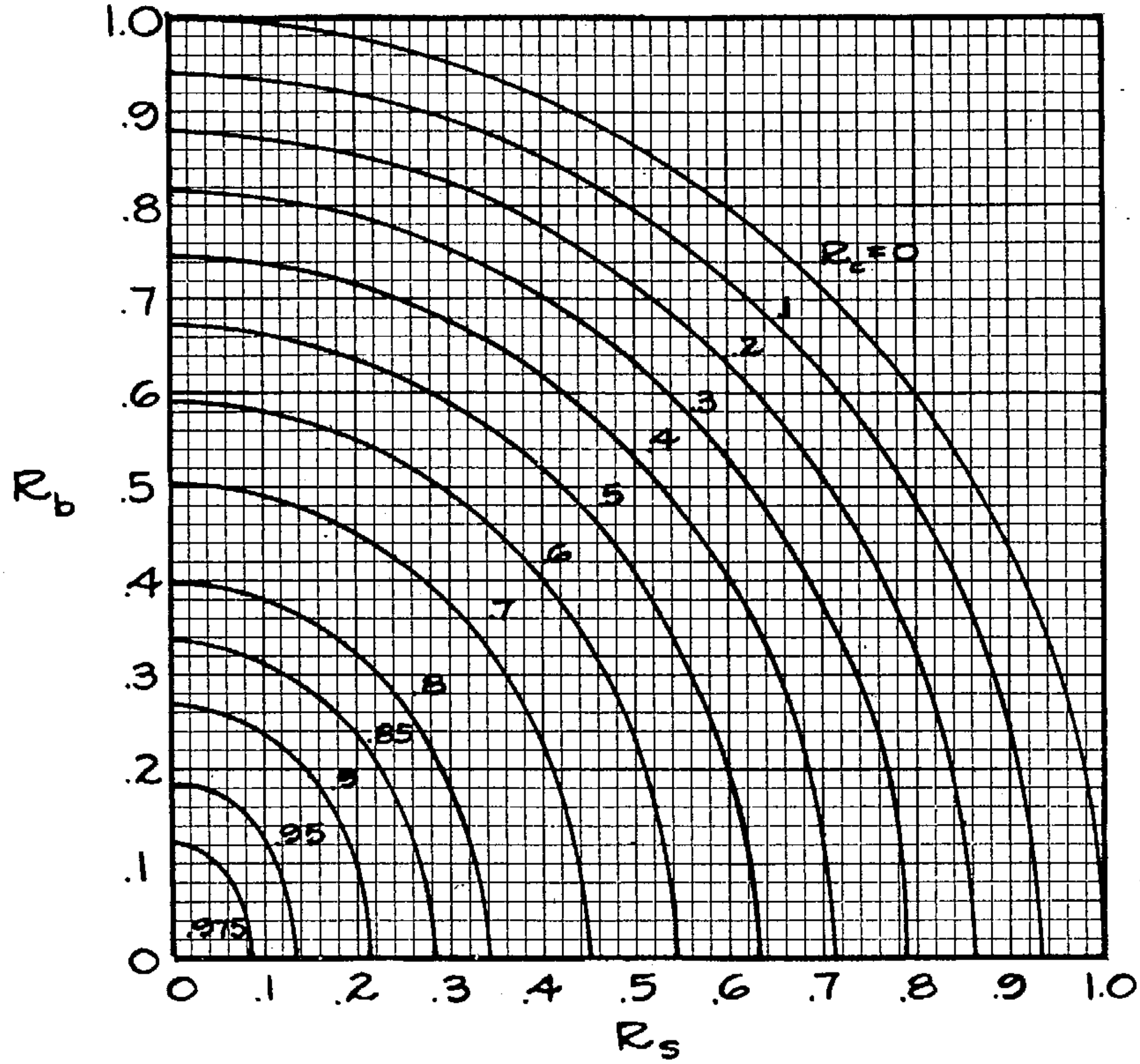


FIG. 1-7 INTERACTION CURVES FOR THE ELASTIC BUCKLING OF FLAT RECTANGULAR PANELS

1.612 Ultimate Strength. It should be noted that flat panels are often used beyond their buckling strength in sheet-stiffeners combinations. The ultimate strength of such combinations is discussed in Secs. 1.711 and 1.721.

## 1.62 CURVED PANELS

1.620 Edge Compression. Curved panels under compression in the directions of their elements can be treated as portions of circular cylinders of the same radius if their edges are adequately stiffened. The buckling stresses of such panels can be determined by the formula of Sec. 1.630.

1.621 Shear. In Ref. 6 the equation for the buckling stress of simply supported curved plates under shearing loads is expressed in the following form:

$$F_{s_{cr}} = KE\left(\frac{t}{b}\right)^2 + K_1E\,\frac{t}{r} \quad \text{------------- (1:48)}$$

Where $t$ = thickness of sheet.

$b$ = arc length of curved edge of plate.

$r$ = radius of curvature of plate.

The first term of the above equation represents the buckling shear stress for a flat plate having the same corresponding dimensions as the curved plate, and the value of the constant K should be taken equal to that for the corresponding flat plate (See Fig. 1-6). The second term of the equation gives the additional stress that can be carried due to the curvature of the plate. In view of existing data it appears that the value of $K_1$ can safely be assumed to be .10.

1.622 Combined Loadings. There is very little information on the buckling of curved panels under combined loadings. It is quite likely, however, that the results of tests for such conditions can be represented by general equation 1:37.

1.623 Ultimate Strength. As in the case of flat panels, curved panels are also used beyond their buckling strength in sheet-stiffener combinations. The ultimate strength of such combinations is discussed in Secs. 1.712 and 1.722.

## 1.63 CIRCULAR THIN-WALLED CYLINDERS

1.630 Circular Thin-Walled Cylinders in Compression. The general theory of thin-walled cylinders in compression is summarized in Ref. 7. Although the theoretical equations are complex, for practical purposes a simplified formula of the following form can be used:

$$F_c = K\frac{E}{D/t} \quad \text{------------- (1:49)}$$

For metals, the factor K will depend largely on the accuracy of the construction. It should be noted that the departure of test values from the theoretical values becomes greater as higher diameter-thickness ratios are employed, indicating that initial eccentricities have a pronounced effect on the stress at which complete failure occurs.

1.631 Circular Thin-Walled Cylinders in Bending. In Ref. 8 it is shown that the modulus of rupture in bending (stress at which collapse occurs, computed by ordinary beam theory) can be expressed by a formula similar to Eq. 1:49, in which the factor K is from 30 to 80 per cent higher than for simple compression. This applies only to cases in which pure bending exists, or in which the shear is relatively low. (See Sec. 1.633 for the case of combined transverse shear and bending.)

1.632 Circular Thin-Walled Cylinders in Torsion. In Ref. 9 it is shown that two general types of formulas apply, depending on the length of the tube. Long slender tubes are classed as those for which the quantity $\frac{1}{\sqrt{1-\mu^2}} \times \frac{L^2 t}{D^3}$ is greater than 7.8 for clamped edges, or greater than 5.5 for hinged edges. For most structural materials the following equation will serve for engineering purposes, disregarding end conditions:

$$\frac{L^2 t}{D^3} = 6.0 \qquad (1{:}50)$$

When the value of $\frac{L^2 t}{D^3}$ is less than that given in equation (1:50), the tubes are classed as short and medium length tubes.

(1) Short and medium length tubes. The minimum buckling stress for an average tube is given by Eqs. 4 of Ref. 9. For engineering purposes these equations can be simplified when the value of H (as defined in Ref. 9) is greater than approximately 19. This corresponds to cases in which the quantity $\frac{L^2}{tD}$ is greater than 20 (approximately). In such cases the following formula will apply:

$$F_{st} = K \frac{E}{\left(\frac{D}{t}\right)^{5/4} \left(\frac{L}{D}\right)^{1/2}} \qquad (1{:}51)$$

The minimum value of K to be expected for average tubes will be approximately:

K = .80 (clamped edges).

K = .75 (hinged edges).

(2) Long slender tubes. As indicated above, when the quantity $\frac{L^2 t}{D^3}$ is greater than 6.0, the tube is classed as a long slender tube, in which case the following formula will apply:

$$F_{st} = K \frac{E}{(D/t)^{3/2}} \qquad (1{:}52)$$

The minimum value of K to be expected for average tubes will be in this case approximately 0.70.

1.633 Circular Thin-Walled Cylinders Under Transverse Shear and Bending. The results of tests of duralumin cylinders in combined transverse shear and bending are given in Ref. 10. The results indicated that three cases apply, as follows:

(a) Bending predominating.
(b) Shear predominating.
(c) Intermediate conditions.

In case (a) the failure will take place at a stress approximately equal to that for pure bending. In case (b) it was found that the allowable transverse shear stress appeared to be about 20 per cent higher than the corresponding allowable shear stress for pure torsion. (See Sec. 1.632).

For immediate conditions an approximate theory was established (Ref. 10 pages 8 and 9) which can be expressed in the system described in Sec. 1.424 by the following equation:

$$R_b^2 + R_s^2 = 1.0 \quad \text{- - - - - - - - - - - - - - - - - -} \quad (1{:}53)$$

where: $R_b = \frac{f_b}{F_b}$ ,

$R_s = \frac{f_s}{F_s}$ .

The allowable shear stress $F_s$ would be obtained as for pure torsion and then increased not more than 20 per cent. Equation(1:53)when plotted in the form of $R_b$ vs. $R_s$ yields a circular arc with the center at the origin (See Fig. 1-2, page 1-18). The derivation of this equation was based on assumptions which appear to yield low values and it is quite possible that for a given case the test data would plot outside the circular curve indicated by Eq. (1:53). The limiting case appears to be that in which no intermediate failure would occur, the corresponding curve being a pair of lines representing $R_b = 1.0$ and $R_s = 1.0$.

1.634 Circular Thin-Walled Cylinders in Combined Bending and Compression. Although very few data exist from which any definite conclusions may be drawn, it appears probable that the following equation can be used in the case of combined bending and compression:

$$R_c + R_b = 1.0 \quad \text{- - - - - - - - - - - - - - - - - - - -} \quad (1{:}54)$$

where $R_c$ and $R_b$ are computed in accordance with Secs. 1.630 and 1.631. This equation is discussed in Sec. 1.424 and is shown as a straight line on Fig. 1-2, page 1-18. When the member in question is subject to a major instability failure, as in the case of a long slender tube, the above equation will not necessarily apply.

1.635 Circular Thin-Walled Cylinders in Combined Bending and Torsion. The nature of the exponents in the general equation (1:37) of Sec. 1.424 should be determined by tests of typical sections, when combined bending and torsion exist. From the relatively few test results which are available on this subject it appears that an equation of the form

$$R_b^2 + R_s^2 = 1.0 \text{ - - - - - - - - - - - - - - - - - -(1:55)}$$

will give reasonable results. This equation is plotted on Fig. 1-2, page 1-18. If no tests are made to check this equation, it would be advisable to use the equation:

$$R_b + R_s = 1.0 \text{ - - - - - - - - - - - - - - - - - - -(1:56)}$$

which will safely cover all cases, provided that the values of R are based on safe values for the allowable stresses in simple bending and torsion.

When transverse shear stresses exist in addition to the shear stresses due to torsion, it appears to be a safe procedure to add the values of R for torsion and transverse shear, respectively, in which case equation (1:37) becomes

$$R_b^2 + (R_s + R_{st})^2 = 1.0 \text{ - - - - - - - - - - - - -(1:57)}$$

where $R_s$ corresponds to transverse shear

and $R_{st}$ corresponds to torsional shear.

1.636 Other Combined Loadings. Tests should be made to determine the nature of the curve given by the general equation (1:37).

## 1.64 ELLIPTIC THIN-WALLED CYLINDERS.

1.640 Compression. (No data available).

1.641 Bending. The following summary of tests on duralumin cylinders of elliptic section is taken from page 10, Ref. 11: "For pure bending in the plane of the major axis, the calculated stress on the extreme fiber at failure was greater than the corresponding stress for circumscribed circular cylinders of the same sheet thickness and length. As in the case of circular cylinders, it was found that slight imperfections caused the test results to scatter widely. In view of this scattering, the bending strength must be considered somewhat indefinite and should therefore be estimated after consideration of all the experimental data for both elliptic and circular cylinders. Design values may then be chosen more or less conservatively as desired".

1.642 Torsion. The following information is taken from page 10, Ref. 11. "In torsion, the shearing stress at failure for thin-walled elliptic cylinders was found to be equal to the shearing stress at failure for circumscribed circular cylinders of the same sheet thickness and length. Because buckling of the walls occurred at the ends of the minor axis prior to failure, the shearing stress calculated for the elliptic cylinder must be regarded as analogous to the modulus of rupture and so used in strength calculations".

1.643 Transverse Shear and Bending. The information given in Sec. 1.633 will apply also to elliptic tubes, according to best available data.

1.644 Other Combined Loadings. Tests should be made to determine the nature of the curve given by the general equation (1:37).

## 1.65 THIN-WALLED TRUNCATED CONES OF CIRCULAR CROSS SECTION.

1.650 General. For the computation of the stresses in thin-walled truncated cones it is necessary to modify the formulas for thin-walled cylinders in order to account for the angle $\alpha$ between the elements and axis of the cone. In the unpublished N.A.C.A. tests, from the results of which this section is prepared, $\alpha = \tan^{-1}(1/5)$. The formulas given may therefore be considered to apply provided $\alpha$ does not exceed $\tan^{-1}(1/5)$.

1.651 Compression. On the assumption that the internal compressive stress $f_c$ acts in the direction of the element

$$f_c = \frac{P}{A}\sec\alpha$$

$$= \frac{P}{2\pi rt}\sec\alpha \qquad \text{(1:58)}$$

The allowable compressive stress at any section can be assumed to be the same as that for a circular cylinder having the same value of D/t. See Eq. (1:49), Sec. 1.630.

1.652 Bending. On the assumption that the bending stresses act in the direction of the elements, the stress on the extreme fiber is

$$f_b = \frac{Mc}{I}\sec\alpha$$

$$= \frac{M}{\pi r^2 t}\sec\alpha \qquad \text{(1:59)}$$

The allowable bending stress on the extreme fiber can be assumed to be the same as that for a circular cylinder having the same value of D/t. See Sec. 1.631.

1.653 Torsion. For torsion, the shearing stress in the plane of the skin is given by the formula

$$f_s = \frac{Tr}{J}$$

$$= \frac{T}{2\pi r^2 t} \qquad \text{(1:60)}$$

The allowable shearing stress in torsion can be assumed to be equal to the allowable shearing stress of a circular cylinder having the same values of D/t and L/D. (See Sec. 1.632)

1.654 Transverse Shear and Bending. On the assumption that the bending stresses act in the direction of the elements, a portion of the shear V is resisted by the bending stresses. Stated in more general terms, a moment M on a truncated cone of circular cross section induces a shear $V_i$ where

$$V_i = -\frac{M}{r} \tan \alpha$$

In combined transverse shear and bending the effective shear V' is, therefore

$$V' = V + V_i$$
$$= V - \frac{M}{r} \tan \alpha \quad \text{------------------} (1{:}61)$$

The effective transverse shear causes a shearing stress $f_s$ in the plane of the skin at the neutral axis that is given by the formula

$$f_s = \frac{V'Q}{Ib}$$
$$= \frac{V'}{\pi r t} \quad \text{------------------} (1{:}62)$$

The allowable strength in combined transverse shear and bending can be computed in the same manner as for circular cylinders. See equation 1:53, Sec. 1.633.

1.655 Other Combined Loadings. Tests should be made to determine the nature of the curve given by the general equation 1:37.

## 1.7 STIFFENED THIN-WALLED SECTIONS

### 1.70 GENERAL.

In computing the strength of thin-walled sections which are stiffened in axial and circumferential directions, consideration should be given to the true distribution of the internal stresses. This will be affected by a number of items, of which the following are of special importance:

(1) Behavior of the metal covering in compression and as a shear web, including the effects of wrinkling.

(2) Effects of doors, windows, and similar cut-outs.

(3) Effects of "shear-lag".

The ultimate strength of stiffened thin-walled structures under compressive load (due to compression or bending) is dependent on the type of failure which occurs. For instance, the longitudinal stiffeners may fail as columns between the transverse stiffeners; or a more general type of instability failure, involving bending of both the longitudinal and transverse members, may occur. Information on the latter type of failure is contained in Ref. 12.

The allowable stresses for stiffened panels acting as component parts of a thin-walled structure are usually very closely connected with the methods of stress analysis leading to the determination of the internal stresses. Since such methods are not included in the scope of this handbook, the discussion presented herein will be confined to general remarks on the methods of predicting the allowable loads of stiffened panels. In general, tests should be conducted on typical panels to verify the method of computation used.

### 1.71 EDGE COMPRESSION

1.710 General. The general theory of the strength of sheet-stiffener combinations under edge compression is based on the fact that when a compressive load is applied to a stiffened panel in the direction of the stiffeners, the thin plate will be subjected to a stress of varying intensity. The intensity of this stress will tend to be a maximum at the stiffeners and a minimum midway between stiffeners.

1.711 Flat Panels. For convenience it is assumed that the sheet, which is under a variable stress, can be replaced by effective widths of sheet cooperating with each stiffener and subjected to the same stress as the stiffener. These effective widths are shown as the shaded portions of the sheet in Fig. 1-8, Page 1-41, the rest of the sheet being considered as ineffective. The total effective width for any stiffener is made up of the increments $w_1$, $w_2$, etc., which are dependent on the type and location of the stiffener. The general equation for computing the increments of effective width can be expressed as follows:

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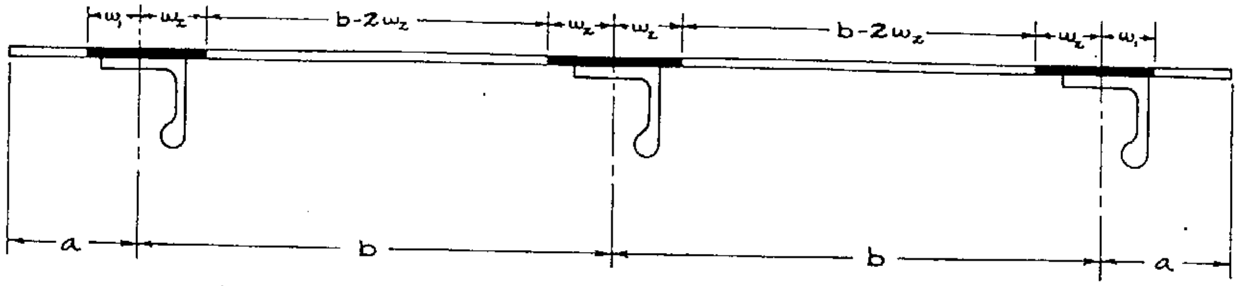
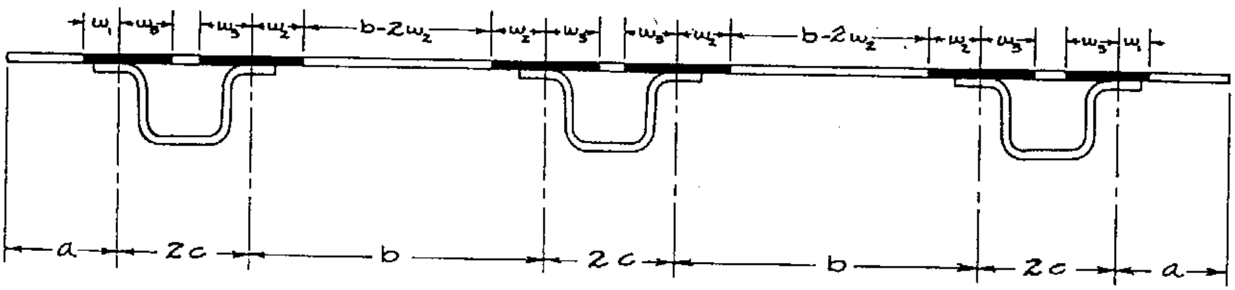
$$w = Ct\sqrt{E/f_c} \qquad (1:63)$$

where t is the thickness of the sheet and $f_c$ is the stress in the stiffener. Values of the constant C for different types of sheet-stiffener combinations




(a) PANEL WITH STIFFENERS HAVING A SINGLE LINE OF CONNECTION TO THE SHEET




(b) PANEL WITH STIFFENERS HAVING A DOUBLE LINE OF CONNECTION TO THE SHEET

FIG. 1-8. EFFECTIVE WIDTHS OF SHEET FOR SHEET-STIFFENER COMBINATIONS using aluminum alloys are given in Table I-2. It should be noted that these values apply when the sheet remains unbuckled between the points of connection to the stringer and when the quantity $\sqrt{\frac{E}{f_c}} \cdot \frac{t}{b}$ does not exceed 0.4. Information on the buckling of the sheet between the points of connection to the stiffener is contained in Ref. 14.

TABLE I-2. EFFECTIVE WIDTH CONSTANTS FOR ALUMINUM ALLOY SHEET

| | Stiffener | w | C for Eq. 1:63 | $w_{max}$ (refer to Fig. 1-8) |
|---|---|---|---|---|
| Stiffeners Having Single Line of Connection | Edge | $w_1$ | .60 | a |
| | " | $w_2$ | .85 | b/2 |
| | Intermediate | $w_2$ | .85 | b/2 |
| Stiffeners Having Double Line of Connection | Edge | $w_1$ | .60 | a |
| | " | $w_2$ | .85 | b/2 |
| | " | $w_3$ | .85 | c |
| | Intermediate | $w_2$ | .85 | b/2 |
| | " | $w_3$ | .85 | c |

NOTE: The above constants apply when the sheet remains unbuckled between rivets and when $\sqrt{\frac{E}{f_c}} \cdot \frac{t}{b}$ does not exceed 0.4.

Plate-stiffener combinations may fail by instability in bending or, if the stiffener sections are open, by twisting instability as described in Ref. 2. Local instability failures can occur at free edges or between widely spaced rivets. The support afforded by the sheet will tend to stabilize the stiffeners and may have an appreciable effect on the type of failure involved. For these reasons it generally is necessary to establish allowable stresses for sheet-stiffener combinations by reference to test data for specimens closely simulating those used in the actual structure. (See Refs. 21, 22, and 23.)

It should be noted that the radius of gyration of a composite column consisting of a stiffener and its effective width of sheet may be less than that for the stiffener alone. The extent to which this effect should be considered must be based on the experience of the designer.

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1.712 Curved Panels. The strength of a curved sheet-stiffener combination can be estimated by adding to the allowable load computed in accordance with Sec. 1.711, an additional load due to curvature. This additional load can be determined by assuming that the strips of sheet of width $b - 2w_2$ (See Fig. 1-8) become effective in carrying the following stress:

$$F_c = \frac{.25E}{r/t} \text{ - - - - - - - - - - - - - - - - - - - (1:64)}$$

where r is the radius of curvature of the panel and t is the thickness of the sheet.

## 1.72 SHEAR

1.720 General. If the stiffeners break up the sheet into relatively small rectangles or squares, the methods outlined in Secs. 1.610 and 1.621, or similar methods, can be used to determine the stress at which buckling will take place under simple shear loading. If the sheet buckles at a relatively low stress it will continue to resist shearing loads through the formation of a diagonal tension field. The ultimate strength of the combination will then depend on the ability of the stiffeners to carry the loads imposed by the diagonal tension field, and on the strength of the thin web in tension.

1.721 Flat Panels. Information on the subject of flat panels forming fully developed diagonal tension fields is contained in Ref. 15. Intermediate cases may exist in which the web will resist part of the shear load in pure shear and the remainder through the formation of tension fields. The loads in the stiffeners will then be reduced accordingly. In this latter case the determining factors are the magnitude of the normal compressive stress which can be carried by the web in its buckled state and the effectiveness of the stiffeners in reducing the amount of buckling. Consult Ref. 16 for further information on this subject.

1.722 Curved Panels. The behavior of curved webs under loads exceeding the buckling load is discussed in Ref. 6.

## 1.73 CORRUGATED SECTIONS

No general theory is available for corrugated thin-wall sheets subjected to various forms of loading. Test data are presented under the chapters on the appropriate materials.