Simplified Wing Stress Analysis Of A Strut-Braced Monoplane

The following references have been used in summarizing the data applied in this article:

No. 1—Stress Analysis of Commercial Aircraft (1928), by A. Klemin;

No. 2—Elementary Airplane Structural Analysis by Graphic Methods, (1938) by J. P. Eames;

No. 3—Air Service Information Circular No. 520, Stress Analysis of Lieut. Phillip's "Alouette" Biplane;

No. 4—Procedure Handbook for Aircraft Stress Analysis (1940), by Nye-Hamilton-Eames;

No. 5—Aircraft Structures (1950), by D. J. Peery;

No. 6—Aircraft Design (1954), by K. D. Wood;

No. 7—ANC-18, Design of Wood Aircraft Structures;

No. 8—Civil Aeronautics Manuals, Volume II, (incl. supplements to May 1, 1958).

The easiest way to assimilate something new is by reference to an example. Hence, the methods of analysis used in this report will be exemplified by a wing analysis of the Corben C-1 "Baby Ace." Most EAA members are familiar with this design. Another reason for selecting this airplane is its general similarity to the design of a majority of homebuilts, namely, a strut-braced, single bay, rectangular wing monoplane. The analysis of this design can be used by substituting values applying to one's own design and thus solve for stresses and design components which will be adequate in strength.

This article should make it possible, when applied to a similar design, to determine the maximum stresses to be expected under various conditions of loading and to compute sizes of members to withstand such loads. Of more importance, possibly, is the consideration of what it will **NOT** enable you to do:

A—It will **NOT** provide a method of analysis which will guarantee FAA approval from the analysis alone, without static or sand-bag proof loading, - - and

B—It does **NOT** attempt refinements of procedure enabling construction of the highest efficiency to stem from its application.

A simplified stress analysis considers only that portion which is absolutely essential in calculating the requirements for a safe structure. Nothing else is possible in a short article of this type.

DESIGN DATA

It is usual in discussing methods of stress analysis to review basic mechanics and compare the analytical and graphical methods of solution. However, it was felt that this report would be more concise and of more immediate use to the amateur constructor by commencing the actual analysis and referring to the type of solution by example.

Actual wing analysis should begin by listing pertinent data applying to the design in question. This information is listed in TABLE 1 for the "Baby Ace." The source of some of this information will be covered in appropriate notes following the table:

TABLE 1

THE CORBEN C-1 "BABY ACE"

(Modified version by Paul H. Poberezny)

Airfoil section	Clark "Y"
Gross weight, (Wg)	828 pounds
Weight of wings, (Ww)	123 pounds, (See Note 1)
Net weight, (Wg-Ww)	705 pounds
Total wing span, (S)	25 ft., 9 in. (309 in.)
Length of lift strut bay	95 in. (See Note 2)
Length of overhang	59.5 in. (See Note 2)
Chord of wing	54 in.
Incidence	1½ deg.
Area, (incl. ailerons)	112.3 sq. ft.

Location of spars from the leading edge:

Front spar, in inches—8.000 in.—
in % of chord—14.8%

Rear spar, in inches—38.375 in.—
in % of chord—71.0%

Center of pressure in % of chord, (See Note 3):

PHAA, (Positive high angle of attack condition)	24%
PLAA, (Positive low angle of attack condition)	51%
NLAA, (Negative low angle of attack condition)	24%

Load factors, (See Note 4):

PHAA—4.5
PLAA—4.5
NLAA—2.0

Ratio of chord to beam components, (See Note 5):

PHAA— —.30
PLAA— +.15
NLAA— 0

Note 1—The required weights must be computed as closely as possible from comparison with similar airplanes or from average figures. An average weight schedule for wings which have been constructed in wooden designs are:

Lightly loaded biplanes—.8 to 1.0 lb. per sq. ft.
Heavily-loaded biplanes—1.0 to 1.2 lb. per sq. ft.
Semi-cantilever monoplanes—1.1 to 1.3 lb. per sq. ft.
Full-cantilever monoplanes—1.2 to 1.5 lb. per sq. ft.

Metal wings for each type would average .1 to .3 lb. per sq. ft. increase over the above figures. In this design, 1.1 lbs. per sq. ft. will be used to compute wing weight.

Note 2—95 inches is approximately the mean of the actual lift strut bay dimensions for the front spar, (actual—94.66 in.) and the rear spar, (actual—95.25 in.), while 59.5 in. is close to the mean of the actual overhang for the front spar, (actual—59.84 in.) and the rear spar, (actual—59.25 in.). Use of the approximate mean values will not materially affect results of the final computations, but will simplify the work.

Note 3—The current method most commonly used is to compute the loading based on the elastic axis. However, this is a refinement such as referred to in sub-paragraph B on the first page. The method used here, (center of pressure location) uses values obtained from airfoil wind tunnel data from sources referred to in our report No. 1, such as NACA Technical Report No. 824, etc. For the PHAA and NLAA conditions the C.P. is considered to be at the most forward location given in the wind tunnel data charts. For the PLAA condition the C.P. is obtained by observing its location at the point where the lift coefficient, (C_L) is 25% of its maximum value. For some of the most commonly used sections, the values following are expressed in % of chord length from the leading edge:

Airfoil	C.P. Location for PHAA & NLAA	C.P. Location for PLAA
Clark Y	24%	51%
Gottingen 398	30%	50%
USA 27	29%	50%
USA 35B	30%	49%
USA 45	27%	38%
NACA M6	25%	40%
NACA M12	25%	40%

Note 4—The load factor to be used is selected by the designer. FAA specifies a minimum positive maneuvering load factor of 4.4 for utility aircraft or 6.0 for aerobatic. The minimum negative maneuvering load factor is .4 times the positive factor for utility and .5 times the positive factor for the aerobatic category. A positive load factor of 4.5 is selected for this design, as it is intended to be non-aerobatic, thus giving a negative load factor of 1.8 (4.5 x .4). However, to simplify computations and err on the safe side, we shall use a value of 2.0.

Note 5—Ratio of chord to beam components are positive (+), when chord loads are applied toward the rear of the drag truss and negative (—), when applied in a forward direction. For the PLAA condition, CAM No. 8 in Appendix B, paragraph .2121 states, "Although no aft acting chordwise loading is specified, the structure should be capable of sustaining aft chordwise loads." It has been the practice for many years to apply the value given, (+.15) for this condition in the case of most all wing sections.

SEQUENCE OF WING ANALYSIS

1—Compute the effective span of the wing.

2—Compute the normal gross beam loads.

3—Compute the normal net beam loads.

4—Compute the normal chord loads.

5—Compute distribution of loads between spars.

6—Compute moments, shears and reactions on each spar.

7—Compute loads in lift struts.

8—Solve for drag truss loads.

9—Summarize final total loads in all members.

1—**Effective Span** is the distance from wing tip to wing tip less any portion covered by the fuselage and an allowance for tip loss. The full span of the wing is not effective because the lift forces are greatly reduced at the tips due to air slipping off the tip portion. The decrease in effective semi-span, to allow for tip loss, is shown in Fig. 1 for the externally braced wing where the distance from the outer strut point to the tip is substantially the same as the wing chord. A length equal to ¼ of the overhang, (.25 L_t) is subtracted from the actual semi-span to obtain the **effective** semi-span, or:

Effective semi-span, (Se) = (95 + 59.5) — (.25x59.5) = 139.6 in.

2—**Normal Gross Beam Load** equals gross weight of the airplane divided by the effective span, or:
Gross beam load, (Wgb) = Wg/Se x 2 = 828/139.6 x 2 = 2.97 lb. per in.

3—**Normal Net Beam Load** equals the gross beam load minus the dead weight of the wings, dead weight of wings, (Wwd) per inch run is computed by dividing weight of wings, (Ww) by the **actual**, (not effective) span, (S), or:

Wwd = Ww/S = 123/309 = .4 lb. per in. run.

Net beam load, (Wn) = Wgb—Wwd = 2.97 — .4 = 2.57 lb. per in. run.

Check on work: 2(Wn x Se) + Ww = Wg

2(2.57 x 139.6) + 123 = 840 lbs.

840 lbs. is 12 lbs. more than 828 lbs., but is satisfactory, as it loads the wing more severely and is, therefore, on the safe side.

4—**Normal Chord Loads** are computed by multiplying the net beam load by the chord/beam ratio:

Flight Condition	Net beam load	Chord/beam Ratio.	Chord load per inch	Load Factor	Design Chord load
PHAA	2.57	—.30	—0.77	4.5	—3.47
PLAA	2.57	+.15	0.39	4.5	1.76
NLAA	2.57	0	—	—	—
Dive	—	—	2.28	1	2.28

For the Dive condition, net weight of the airplane, (705 lbs.) is distributed uniformly along the actual span. No load factor is required, hence, 705/309 = 2.28 lb. per in. run.

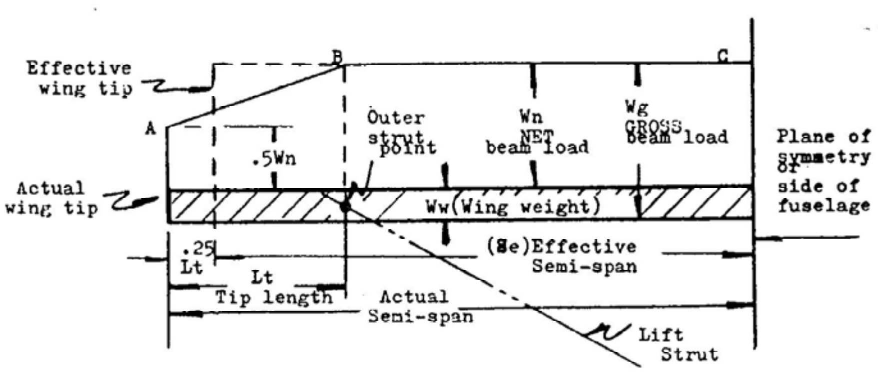

Fig. 1

Simplified Wing Stress Analysis Of A Strut-Braced Monoplane

5—**Distribution of Load Between Spars** in a two-spar design, such as this, is in inverse proportion to spar distance from the center of pressure, at which point the beam load is assumed to act:

Load on front spar:

$$\text{(in \%)}\quad \frac{\text{rear spar location—C.P.}}{\text{rear spar location—front spar location}} \quad \text{or:}$$

for PHAA: 71—24/71—14.8=84% Front spar.

Load on rear spar:

$$\text{(in \%)}\quad \frac{\text{C.P.—front spar location}}{\text{rear spar location—front spar location}} \quad \text{or:}$$

for PHAA: 24—14.8/71—14.8=16% Rear spar.

For PLAA: Front spar: 71—51/71—14.8=35%
Rear spar: 51—14.8/71—14.8=65%

For NLAA: Front spar: 71—24/71—14.8=84%
Rear spar: 24—14.8/71—14.8=16%

TABLE 2
BEAM LOADS PER INCH RUN ON SPARS

Flight Condition	Spar	Net load per in. on Wing	Load Factor	% carried by Spar	Net load per in. run
PHAA	Front	2.57	4.5	84%	9.71
PHAA	Rear	2.57	4.5	16%	1.85
PLAA	Front	2.57	4.5	35%	4.05
PLAA	Rear	2.57	4.5	65%	7.52
NLAA	Front	2.57	2.0	84%	—4.32
NLAA	Rear	2.57	2.0	16%	—0.82

6—**Moments, Shears and Reactions** are computed easiest by placing a unit load on the spar and then increasing the results obtained in direct proportion to the actual net loads per inch on the spar for each condition as in Fig. 2:

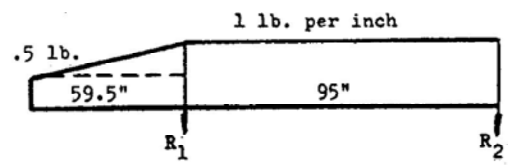

Fig. 2

$M_1=(59.5^2 \text{ x } .5/2)+(59.5^2 \text{ x } .5/2 \text{ x } 3)=885+295$
$=1180$ in. lb.

$M_2=.0$ (Pin jointed).

$S—1=59.5 \text{ x } (1+.5)/2= \quad 44.6$

$S+1= —1180/95—95 \text{ x } ½= \quad —59.9$

$\overline{104.5=R_1}$

$S—2=95 \text{ x } ½—1180/95= \quad 35.1=R_2$

Check: R+R=104.5+35.1=139.6 lb.

Loading: 44.6+95 =139.6 lb. (OK)

Actual moments and reactions are computed in Table 3 by taking "Net load per inch run" from Table 2, for each flight condition and multiplying by the above values at 1 lb. per inch loading for M_1 and R_1

TABLE 3
Calculation of Actual Moments and Reactions

Flight Condition	Spar	Net load per in.	M_1 under 1 lb. per in.	Actual Moment (M_1)	R_1 under 1 lb. per in.	Actual Reaction (R_1)
PHAA	Front	9.71	1180	11,458	104.5	1,015
PHAA	Rear	1.85	1180	2,183	104.5	193
PLAA	Front	4.05	1180	4,779	104.5	423
PLAA	Rear	7.52	1180	8,875	104.5	786
NLAA	Front	—4.32	1180	—5,098	104.5	—451
NLAA	Rear	—0.82	1180	— 968	104.5	— 86

Note that NLAA flight loads are negative as they are acting down, in inverted flight. These loads put compression in the wing struts and as they are fairly long, **this is usually the critical design condition** which will be solved for next.

7—**Lift Strut Loads**

At the point where the lift strut attaches to the wing, the reactions, (R_1) computed in Table 3, pull up in the PHAA & PLAA conditions and push down in the NLAA condition. This puts the struts in tension when pulling up and in compression when pushing down. Strut loads are obtained by multiplying the reaction, (R_1) by the length of the strut, (L) divided by the (V) component as listed in Table 4. The axial load on the spar can be obtained by multiplying the **strut load** by its horizontal component, (H) divided by its length, (L). If the struts slant either away or toward each other so that they are not parallel to the spars they will impose drag and anti-drag loads on the wing. In this design, the struts are out of parallel with the spars by a very small amount but the loads are figured in the last column of Table 5 in order to demonstrate how it should be done.

TABLE 4

Strut	V	H	D	V^2	H^2	D^2	L^2	L
Front	42.25	84.03	.92	1785	7061	.84	8846.84	94.06
Rear	42.25	84.62	1.70	1785	7161	2.89	8948.89	94.60

TABLE 5

Flight Condition	Spar	Reaction R_1	Load in strut (R_1xL/V)	Axial spar load (Strut loadxH/L)	Drag loads D/Lx(Strut load)
PHAA	Front	1,015	2,260T	2,019C	22
PHAA	Rear	193	432T	386C	8
PLAA	Front	423	942T	842C	10
PLAA	Rear	786	1,760T	1,576C	32
NLAA	Front	—451	1,004C	897T	—10
NLAA	Rear	— 86	193C	173T	— 4
DIVE	Front	—451	1,004C	897T	—10
DIVE	Rear	568	1,271T	1,137C	23

NOTE: Negative sign, (—) in last column indicates loads acting forward, or anti-drag loads.

Nose-Dive Condition is illustrated in Fig. 3, where loads on the rear spar are seen to be 1.26 times the loads on the front spar and acting upwards:

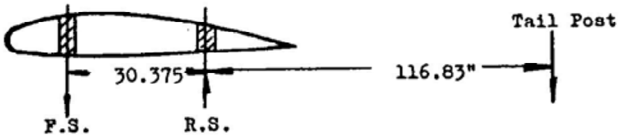

Fig. 3

Load on Rear Spar:

$$\text{F.S.} \times \frac{116.83+30.375}{116.83} = 1.26 \times \text{F.S.}$$

For equilibrium it is evident that the rear spar loads must equal front spar loads plus the load on the tail. Front spar loads for Dive are taken as final design loads on that spar for the NLAA condition.

	NLAA & ND Front Spar	ND Rear Spar (F.S. x 1.26)
Net Load per inch run	—4.32	5.44
M_1	—5,098	6,424
R_1	— 451	568

8—**Drag Truss Loads** consist of two parts, first the design chord loads computed in paragraph 4 of the preceding text, and distributed uniformly along the entire actual wing span, and second the drag and anti-drag loads due to the drag component of the lift struts, computed in Table 5 of the preceding text. The latter is a concentrated load at the point of strut attachment to the spar, whereas, the distributed chord loads are considered concentrated at the panel points, (compression strut location). The load on half a panel at either side of a panel point is considered as applied at the panel point. Loads from wing tip to nearest panel point are considered as applied to that panel point. The solution of the drag truss for PHAA, PLAA and Dive is the same, except that the distributed loads act aft for PLAA and Dive instead of forward, as in PHAA. The drag truss would need to be solved for the NLAA condition in a case where the lift struts are out of parallel with the spars enough to impose a heavy drag load. NLAA will not be solved for in this design due to the low maximum concentrated load of 10 lbs. and no distributed drag load is involved in the NLAA condition.

Drag Truss Solution

The PHAA condition only will be explained in detail as the other conditions are solved by applying the same reasoning. First, sketch the drag truss outline, as in Fig. 4, showing physical dimensions and then obtain lengths of the drag and anti-drag wires either by scaling off the length from the sketch after being careful to draw it accurately to scale or by using the formula of: Wire length equals the square root of the sum of the vertical dimension squared plus the horizontal dimension squared. Note that as each drag bay is a different length, this computation, (or scaling of the length) will have to be made for each bay. Also compute the values of L/V and H/V to obtain the factors used in computing loads in wires and loads in spar sections. Load in wire equals the vertically applied load times the L/V ratio, and load in spar equals the vertically applied load times the H/V ratio. These relationships will be worked out a little later on in the detail drag truss analysis. Next, show the known loads in the correct location and direction and assign letters at panel points and spar junctions to identify members as in Figs. 5, 6 and 7. The concentrated drag loads imposed by the lift struts are obtained from the last column of Table 5; the PHAA condition applying 22 lbs. in an aft direction at the front spar strut connection and 8 lbs. aft at the rear spar strut fitting, as shown in Fig. 4.

The distributed drag loads are computed from the dimensions shown in Fig. 4 and the "design chord load" per inch, listed in the last column of table in paragraph 4 on page 13 of November, '63 SA. As an example, at panel point A in Fig. 5, the distributed load of 113.6 lbs. is obtained by taking the tip overhang of 9.75 in. plus half the 46 in. dimension of the adjacent panel, or 23 in., equaling 32.75 in., times the load of —3.47 lbs. per inch run obtained from paragraph 4, page 13 of November SA, for the PHAA condition.

Now, let's begin the solution of drag truss loads for individual members comprising the truss. Begin at panel point B in Fig. 5 and note that the 113.6 lb. load has to pass directly into member B-A, since member B-C is a wire and cannot take compression and spar section B-D is at right angles to the load and therefore cannot take it either. At panel point A we have 113.6 lbs. pushing up. The wire A-D must pull down to maintain equilibrium in the vertical plane. The load is, therefore, 113.6 x L/V or 113.6 x 1.82 giving 207 lbs. tension in wire A-D. This exerts a horizontal force at panel point A which is balanced by the spar section A-C pushing outward. The force necessary is 113.6 x H/V or 113.6 x 1.52 giving —173 lbs. The minus sign, (—) is used to indicate compression.

Considering panel point D we have the vertical force of 113.6 lbs. exerted by wire A-D in addition to the vertical panel load of 155.1 lbs., equaling a total of 268.7 lbs. This vertical load is direct compression in strut member D-C. The horizontal component of the wire A-D, 173 lbs. is taken in tension by spar section D-F. At panel point C we have the vertical load of 268.7 lbs. pushing up, partially offset by the 22 lbs. concentrated strut drag load pushing down, so 268.7 — 22 gives 246.7 lbs. This is the vertical load which must be taken by wire member C-F. Load in wire is 246.7 x 1.87 giving 462 lbs. The horizontal component of the wire is 246.7 x 1.58 giving 390 lbs., which must be resisted in compression by spar section C-E. But this member must also resist the compression force of 173 lbs. transmitted by spar section A-C, so that total compression on spar section C-E is 173 plus 390 giving —563 lbs.

Considering panel point F, the horizontal component of the tension in wire C-F exerts a tension of 390 lbs. in spar member F-H, to which must

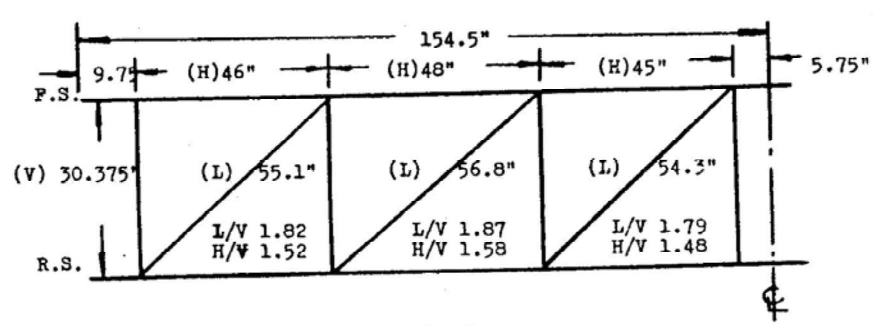

Fig. 4

be added the tension transmitted by spar member D-F of 173 lbs., or a total of 563 lbs. The vertical force of 246.7 lbs. exerted by wire C-F must be added to the panel point load of 161.4 lbs. to give a total of —408.1 lbs. taken by strut member F-E in compression. At panel point E the vertical force 408.1 lbs. is taken in tension by wire E-H, equal to 408.1 x 1.79 or 731 lbs. The horizontal component due to the wire, is 408.1 x 1.48 giving 604 lbs., to which must be added the 563 lbs. compression transmitted from spar section C-E to give a total of 1167 lbs. compression in spar section E-G. This is held in equilibrium by the reaction of —1167 lbs. provided by the front spar root of the opposite wing panel.

Considering panel point H, the horizontal component of the tension in wire E-H exerts a tension of 604 lbs. which must be added to the 563 lbs. tension transmitted by spar section F-H to give 1167 lbs. tension at panel point H, balanced by an equal tension in the rear spar root of the opposite wing panel. The vertical component of 408.1 lbs., of wire E-H added to the panel point load of 98 lbs. gives 506.1 lbs. compression in strut member H-G which is held in equilibrium by an opposite reaction of 506.1 lbs. provided by the cabane attachment structure of the fuselage.

The PLAA and Dive conditions are solved in the same manner and the values have been shown on Figs. 5, 6 and 7, respectively.

9—**Summary of Total Drag Loads** should now be shown in table form such as Table 6, and the maximum loads selected to be the design loads as listed in the last column.

TABLE 6
Summary of Drag Truss Load

Member		PHAA	PLAA	Dive	Design Load
Drag Wires	B-C	0	105	136	136
	D-E	0	342	365	365
	F-G	0	474	539	539
Anti-drag Wires	A-D	207	0	0	207
	C-F	462	0	0	462
	E-H	731	0	0	731
Compression Struts	A-B	—114	—58	—75	—114
	C-D	—269	—151	—172	—269
	E-F	—408	—265	—301	—408
	G-H	—506	—315	—366	—506
Front Spar	A-C	—173	0	0	—173
	C-E	—563	88	114	—563
	E-G	—1167	377	422	—1167
Rear Spar	B-D	0	—88	—114	—114
	D-F	173	—377	—422	—422
	F-H	563	—769	—868	—868

NOTE: (—) indicates compression.

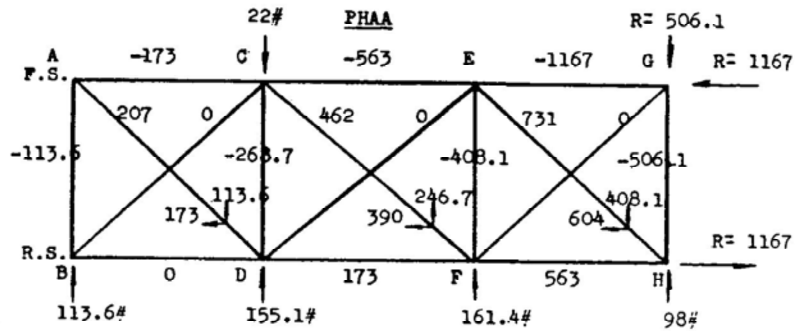

Fig. 5

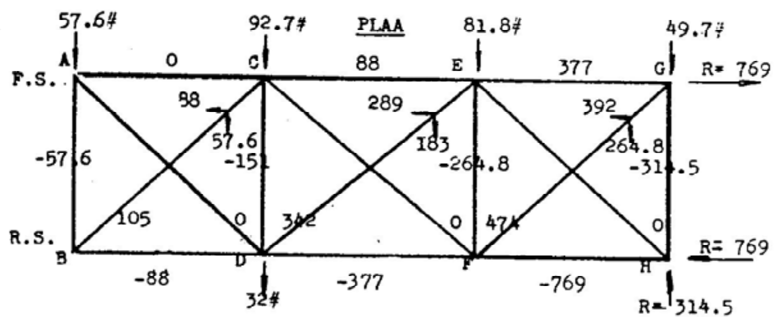

Fig. 6

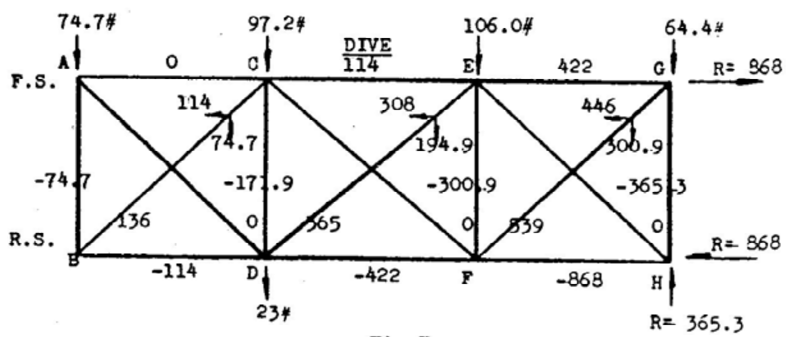

Fig. 7

Simplified Wing Strut Analysis Of A Strut-Braced Monoplane

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THIS REPORT is a continuation of Part 2 in the December, 1963 issue of *SPORT AVIATION*, in that it investigates design procedures for the structural members analyzed for stresses in that report. The references consulted in the preparation of this paper are as follows:

No. 1—Stress Analysis of Commercial Aircraft (1928) by A. Klemin;

No. 2—Procedure Handbook for Aircraft Stress Analysis (1940) by Nye-Hamilton-Eames;

No. 3—ANC-5, Strength of Metal Aircraft Elements (1955) by GPO.

No. 4—ANC-18, Design of Wood Aircraft Structures (1951) by GPO.

No. 5—Airplane Structures, Vol. 1, 3rd edition (1943) by Niles.

Design of Lift Struts

In the previous analysis of the wing, the stresses which were first obtained in complete form were those imposed on the external lift struts. We will, therefore, consider first the design procedure for these struts. In the following table, "computed stresses" are the most severe of those listed on Table 5 of the preceding article.

The Civil Aeronautics Manuals, Vol. 11, in CAM No. 3, page 27, specifies that a safety factor of 1.5 be imposed, in addition to the load factor, to take care of variation in individual lots of material from the test values given for strength. This has been done to obtain the values indicated in the last column of Table A, and will be applied to other stress values obtained from the previous article.

A member carrying compression is customarily referred to as a "column", and since the method of end attachment affects the problem considerably, the first step is to determine the coefficient of fixity, referred to as "c." "c" equals 1 for pin-ended columns, and "c" equals 2 for restrained columns such as welded sections of tubing in a fuselage structure.

The length of the column is extremely important and the ratio L/p is used to designate a column as "short" or "long", based on what is called the critical L/p value. This critical value is listed in references No. 1, No. 2 and No. 3, for different kinds of materials. If the L/p value is below the "critical" value, it is a **short** column. If above, it is considered a **long** column. L is the length of the column in inches and p, (the letter "rho" in the Greek alphabet) symbolizes the radius of gyration.

The radius of gyration can be computed as $p = I/A$, where, I = moment of inertia of the column section and A = cross-sectional area of the column in square inches. Practically all of these values can be obtained from tables in various handbooks or references, No. 2, No. 3 or No. 5 and therefore do not have to be computed in practice. The ratio L/p should not exceed 150 and should be kept as small as possible.

"Local instability" is failure by crushing of the walls of the column. This occurs when the wall thickness of the column or tube is too small a value for its diameter. Tubes do not need to be investigated for this condition if the diameter (D), divided by the wall thickness (t), is less than 50.

The sequence of operations necessary to select a proper size tube is as follows:

1—Arbitrarily choose a standard size tube.
2—Determine whether it is a long or short column.
3—Determine if column should be investigated for local instability, (D/t over 50).
4—Solve for allowable stress.
5—Solve for actual stress.
6—Determine the margin of safety. If margin of safety, (M.S) is negative in value, choose a stronger tube and repeat the calculations. If it has an excessively positive margin, choose a smaller tube in order to save weight and recalculate.

Application of the foregoing to the "Baby Ace" design follows:

Step No. 1—To check for the proper size of streamline tube to be used for a front strut, we will investigate the 87 in.—SAE No. 4130 steel tube originally called for, and having a 2.697 in. major axis, a 1.143 minor axis, and a wall thickness of .065 in. From reference No. 2 or No. 3 we find p of major axis = .4062 and L/p = 87/.4062 = 214. An L/p of 214 is greatly in excess of 150 which should not be exceeded. However, this plane has short jury struts connecting the long lift struts to the spar at a point 35 in. inboard from the outer strut point so this restraint makes it possible to compute the value of L as 52 in. in place of 87 in. If we substitute this value in our L/p equation we get: 52/.4062 = 128, which is obviously satisfactory.

Step No. 2—As the critical L/p value for No. 4130 steel from reference No. 1, No. 2 or No. 3 is 91, this is considered a long column.

Step No. 3—As D/t = 2/.065 = 30.7, this tube will not need to be investigated for local instability as the D/t value is below 50. (The D = 2 value is the equivalent round tube diameter from which the streamline tube is formed, also listed in references No. 1, No. 2 or No. 3).

Step No. 4—The allowable stress (Fc) for long columns is equal to:

$$Fc = \frac{286x\ 10^6}{(L/p)^2}$$

To err on the safe side, we shall consider L/p = 214, the original value obtained and ignore the action of the jury struts, hence: $Fc = 286{,}000{,}000/(214)^2 = 286{,}000{,}000/45{,}796 = 6135$ psi.

Step No. 5—The actual stress (fc) is equal to P/A where P is the design stress and A is the area of the tube. Hence, 1506/.3951 = 3812 psi.

Step No. 6—Margin of safety is Fc/fc—1 or, 6135/3812—1 = .609 and as this is a positive value, it is satisfactory.

(Continued on next page)

TABLE A

Strut	Type of Stress	Condition	Computed Stress in #	DESIGN STRESS (Comp. stress x 1.5)
Front	Compression	NLAA	1,004	1,506
Front	Tension	PHAA	2,260	3,390
Rear	Compression	NLAA	193	289
Rear	Tension	PLAA	1,760	2,640

TABLE B
DESIGN OF DRAG TRUSS MEMBERS

Member		Computed Stress	Design Stress	Material	Fc	fc	M.S.
Compression Strut	A-B	—114	—171	¾"x¾" Spruce	750 psi	152 psi	3.93
Compression Strut	C-D	—269	—404	¾"x¾" Spruce	750 psi	359 psi	1.09
Compression Strut	E-F	—408	—612	¾"x¾" Spruce	750 psi	544 psi	0.38
Compression Strut	G-H	—506	—759	¾"x¾" Spruce	750 psi	675 psi	0.11
Anti-drag Wire	A-D	207	311	#6-40 Tie rod	1000#	311#	2.21
Anti-drag Wire	C-F	462	693	#6-40 Tie rod	1000#	693#	0.44
Anti-drag Wire	E-H	731	1097	#6-40 Tie rod	1000#	1097#	—0.09
Drag Wire	B-C	136	204	#6-40 Tie rod	1000#	204#	3.90
Drag Wire	D-E	365	548	#6-40 Tie rod	1000#	548#	0.82
Drag Wire	F-G	539	809	#6-40 Tie rod	1000#	809#	0.24

Simplified Wing Stress . . .

Performing the same steps for the rear lift strut, which was originally designed as an 89 in.—SAE No. 4130 steel streamline tube having a major axis of 1.686 in., a minor axis of .714 in. and a wall thickness of .049 in., we compute:

Step No. 2—p of major axis = .2509, L/p = 89/.2509 = 355. This, again, is greatly in excess of 150, so we apply the same reasoning to use of the jury struts, and recompute for a length of 52 in., or 52/.2509 = 207. This is still excessive as an L/p ratio, but the one factor allowing its use is the extremely low design compression load of 289 lbs.

Step No. 3—D/t = 1.25/.049 = 25.5, well under 50.

Step No. 4—Again we shall err on the safe side by using the original L/p value of 346, thereby giving:
$Fc = 286{,}000{,}000/(355)^2 = 286{,}000{,}000/126{,}000 = 2270$ psi.

Step No. 5—fc = 289/.1849 = 1562 psi.

Step No. 6—M.S. = 2270/1562—1 = .453.

We now must check to see if the struts will withstand maximum tension loads. The allowable tensile yield stress for SAE No. 4130 steel is 75,000 psi, hence, $F_t = P \times A$, which, for the front strut = 75,000 x .3951 = 29.632 lbs. As our design stress is only 3,390 lbs., this tube is well over strength in tension. (M.S. = 29,632/3,390—1 = 7.74). Rear strut computes as: 75,000 x .1849 = 13,867 lbs., compared with a design stress of 2,640 lbs., again showing an over strength condition in tension. (M.S. = 13,867/2,640—1 = 4.25).

Compression Struts

Step No. 1—Compute the L/p value: For a square or rectangular section, p = .288d where d is the shortest side, hence, .288 x .75 = .216 and L = 29.625 in. Therefore, L/p = 29.625/.216 = 137.

Step No. 2—Solve for allowable stress, (Fc): References No. 1 and No. 4 contain Figs. 41 and 2-6, respectively, which give allowable column stresses in psi for solid wood columns. Referring to ANC-18, (Reference No. 4), Fig. 2-6, we find that a spruce column with an L/p value of 137m has an allowable column stress value of 750 psi.

Step No. 3—Solve for actual stress, (fc): The area of a ¾ in. square strut is .5625 sq. in., hence, fc = P/A or, in the case of compression strut A-B: 171/.5625 = 304 psi. However, as there are double struts at each location, the actual stress on each ¾ in. strut is half the computed amount, or 152 psi. The other three strut locations are computed in the same manner and listed in the sixth column of Table B.

Step No. 4—Compute Margin of Safety: In the case of strut A-B, M.S. = 750/152—1 = 3.93. The remaining struts have their M.S. computed in same manner.

Anti-Drag and Drag Wires

As these members are in pure tension, it is only necessary to look up their rated strength, which can be done in Reference No. 1, Table 10 or Reference No. 2, Tables 49 and 50, giving an allowable tensile load, (F_t) of 1,000 lbs. The margin of safety is computed as for the compression struts, or: $F_t/f_t - 1$ = M.S. These values are listed in column 7. Anti-drag wire, E-H, has a negative margin of safety, which means the next larger size should be specified to meet the requirements of this stress-analysis and give a positive M.S. Note, however, that we arbitrarily selected 4.5 as our load factor for this example although 4.4 is all that the FAA requires. Hence, if the 4.4 factor had been used for PHAA, the computed stresses would be low enough to provide a positive M.S. Therefore, this wire is no doubt strong enough as originally computed by the designer.

Design of Spars

When a spar is subjected to combined bending and compression, the stress at any section is the sum of the stresses due to bending and compression. When a spar deflects under bending, the compressive end load increases the bending stress. As it is stressed more it deflects more, and the end load times the additional deflection increases the stress and deflection still more, until a point of equilibrium is reached. To determine the bending stress at this point of equilibrium we have to use the so-called "Precise Formula."

Before the "Precise Equation" can be solved, it will be necessary to know the moment of inertia, (I) of the spar cross-section. Therefore, a spar section must be designed which will fit the wing profile at the proper location and have a large enough value of I. This is best estimated from previous experience, otherwise a guess must be made and the precise equation solved after which another attempt must be made if the stress obtained is too great.

We will now examine the spar properties of the sections in this design. The front and rear spars have the dimensions shown in Fig. 1.

In applying the precise equations to determine maximum moment in the span, M_{1-2}, the **average** axial drag load in the bay is used. Later, in designing the cross-section to take the axial loads as listed in Table F, the true axial load at each cross-section will be used.

Computations from here on will be made with a 10 in. slide rule so will only have that degree of accuracy possible to attain on such a rule.

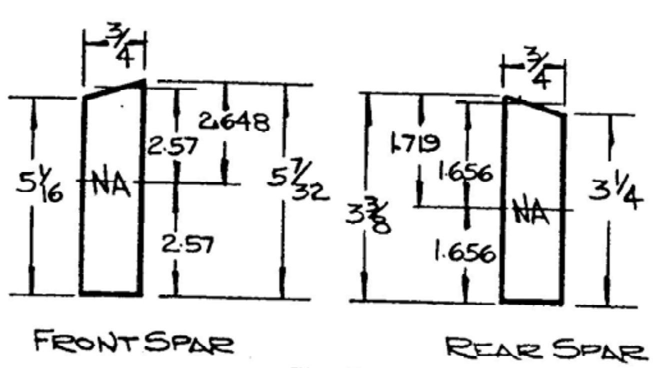

Fig. 1

TABLE C
AXIAL LOADS ON SPARS

Flight Condition	Spar	Average due to Drag	Due to Lift	Total
PHAA	Front	—865	—2,019	—2,884
	Rear	368	— 386	— 18
PLAA	Front	233	— 842	— 609
	Rear	—573	—1,576	—2,149
DIVE	Front	268	897	1,165
	Rear	—645	—1,137	—1,782

Normally, this will not be in error more than 1/10 of 1 percent, which is deemed to be accurate enough for these types of engineering calculations.

The moment of inertia, (I) of a rectangle is: $bd^3/12$. Therefore, I for the front spar is: $.75 \times (5.140)^3/12 = .75 \times 135.7/12 = 8.480$ in. For the rear spar: $.75 \times (3.312)^3/12 = .75 \times 36.329/12 = 2.270$ in. Area of front spar $= b \times d = .75 \times 5.140 = 3.855$ sq. in. Area of rear spar $= .75 \times 3.312 = 2.484$ sq. in.

TABLE E
COMPUTATION OF MAXIMUM MOMENT IN SPAN

Computation	Front Spar—PHAA
Design load, (w)	9.71
$j^2 = EI/P$	3,800
wj^2	36,898
M_1	11,458
$D_1 = M_1 - wj^2$	—25,440
M_2	0
$D_2 = M_2 - wj^2$	—36,898
L/j (in radians)	1.542
L/j (in degrees)	88.36°
Cos L/j	.029
D_1 Cos L/j	—738
$D_2 - D_1$ Cos L/j	—36,160
Sin L/j	.999
D_1 Sin L/j	—25,414
Tan X/j $= D_2 - D_1$ Cos L/j / D_1 Sin L/j	1.422
Tan^2 X/j — 1	3.022
Sec X/j $= \sqrt{\text{Tan}^2 X/j - 1}$	1.738
D1 Sec X/j	—44,214
$M_{1-2} = D_1$ Sec X/j — wj^2	—7,316

LOCATION OF MAXIMUM MOMENT IN SPAN

Tan X/j = 1.422 X/j = 54.9° Converted to radians: 54.9/57.3 = .958 radians. j = 61.6 (from Table D). Therefore, X = X/j x j or, .958 x 61.6 = 59.01 inches inboard from outer strut point.

TABLE F
COMPUTATION OF STRESSES IN FRONT SPAR

Item	Front Spar — PHAA M_1 out	M_1 in	M_{1-2}
Moment (M)	11,458	11,458	7,316
y/I (y = 2.648, I = 8.480)	.3122	.3122	.3122
$f_b = My/I$ (bending stress)	3,577	3,577	2,284
P (Axial load)	—173	—2,582	—3,186
A (Area of spar)	3.855	3.855	3.855
$f_c = P/A$ (Compression stress)	—45	—670	—827
$f_t = f_b + f_c$ (Required stress)	3,622	4,247	3,111
Modulus of Rupture	9,400	9,400	9,400
L/p (46/2.186 & 95/2.186)	21.0	43.5	43.5
f_b/f_t	.99	.84	.73
F_t (total allowable stress)	9,300 psi	8,600 psi	8,000 psi
f_t (total required stress)	3,622 psi	4,247 psi	3,111 psi
$F_t/f_t - 1$ (M.S.)	1.56	1.02	1.57

As the lowest margin of safety at the strut point is 1.02, the front spar is definitely satisfactory.

The safest and easiest method of analyzing beams with combined **tension** and bending is to neglect the **decrease** in bending due to the tensile load and investigate only the bending stress. This would apply above, in the case of the front spar in dive condition. However, the load per inch run, as found in a preceding article, is only —4.32 lbs., which is less than half that under the PHAA condition, so will not be investigated. If these spars were of box or I beam construction, with a heavier compression flange on top, it would be advisable to check the strength in the inverted flight or dive condition as the load is reversed in direction and the bottom tension flange might prove to be weak in compression. However, as these spars are of solid, rectangular construction, this reversed loading check will not be necessary. From here on, only the front spar in the PHAA condition will be investigated as an example, to reduce the work of computing.

TABLE D
FRONT SPAR LOADS AND PROPERTIES

Flight condition	PHAA
Total axial load, (P)	—2,884
Moment of inertia, (I)	8.480
EI (E = 1,300,000 for spruce)	11,024,000
$j^2 = EI/P$	3,800
j	61.6
Span (L)	95
L/j (in radians)	1.542
L/j in degrees (57.3 L/j in radians)	88.36°
Load per inch run	9.71

Now about that stupid homebuilt project you've been planning . . .